Lesson 10

Further A-Level Pure Mathematics : Core 1

Matrix Transformations

10.1 Pre or Post Multiply ?

The non-commutative nature of matrix algebra demands care when multiplying both sides of a matrix equation. Suppose that **A**, **B**, and **C** are known matrices, with **X** being the unknown matrix to be found in terms of **A**, **B** and **C** and that,

$$\mathbf{AXB} = \mathbf{C}$$

There is no concept of "Matrix Division", so any idea of dividing both sides by **AB** is not going to end well. However, use can be made of the inverse matrices of the known matrices and the "Do Nothing" Identity matrix **I.**

$$\mathbf{AXB} = \mathbf{C}$$

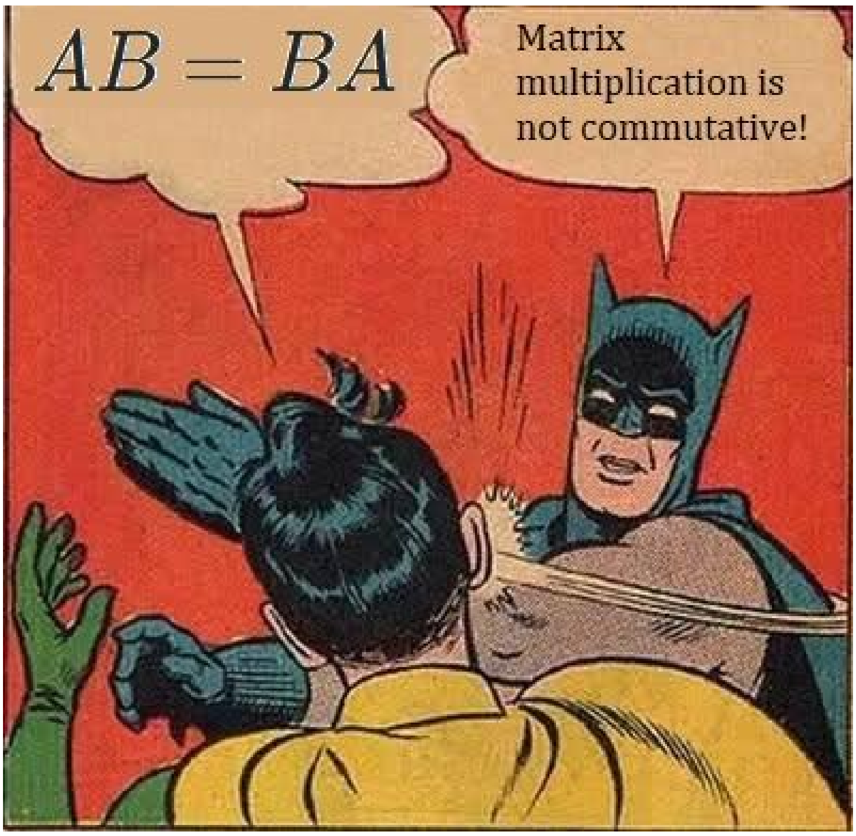
$$\mathbf{A}^{-1}\mathbf{AXB} = \mathbf{A}^{-1}\mathbf{C} \qquad \text{Pre-multiply both sides by } \mathbf{A}^{-1}$$

$$\mathbf{XB} = \mathbf{A}^{-1}\mathbf{C} \qquad \text{As } \mathbf{A}^{-1}\mathbf{A} = \mathbf{I}$$

$$\mathbf{XB}\,\mathbf{B}^{-1} = \mathbf{A}^{-1}\mathbf{C}\,\mathbf{B}^{-1} \qquad \text{Post-multiply both sides by } \mathbf{B}^{-1}$$

$$\mathbf{X} = \mathbf{A}^{-1}\mathbf{C}\,\mathbf{B}^{-1} \qquad \text{As } \mathbf{B}\,\mathbf{B}^{-1} = \mathbf{I}$$

Sometimes "Pre-multiply" is called "Left-multiply" or "Front-multiply"
Similarly "Post-multiply" is also called "Right-multiply" or "Back-multiply"



10.2 A Pre-Multiply, Post-Multiply Example

(i) Given that **ABC** = **I**, prove that $\mathbf{B}^{-1} = \mathbf{CA}$

(ii) Given that $\mathbf{A} = \begin{pmatrix} 0 & 1 \\ -1 & -6 \end{pmatrix}$ and $\mathbf{C} = \begin{pmatrix} 2 & 1 \\ -3 & -1 \end{pmatrix}$, find **B**

Teaching Video : http://www.NumberWonder.co.uk/v9090/10a.mp4 (Part 1)
http://www.NumberWonder.co.uk/v9090/10b.mp4 (Part 2)

<= Part 1 Part 2 =>

[4, 4 marks]

10.3 Exercise

Any solution based entirely on graphical
or numerical methods is not acceptable
Marks Available : 40

Question 1

Further A-Level Examination Question from May 2017, IAL, FP1, Q2 (Edexcel)

$$\mathbf{A} = \begin{pmatrix} 2 & -1 \\ 4 & 3 \end{pmatrix}, \quad \mathbf{P} = \begin{pmatrix} 3 & 6 \\ 11 & -8 \end{pmatrix}$$

(a) Find $\mathbf{A}^{-1}$

[2 marks]

The transformation represented by the matrix **B** followed by the transformation represented by the matrix **A** is equivalent to the transformation represented by the matrix **P**.

(b) Find **B**, giving your answer in its simplest form.

[4 marks]

Question 2

Further A-Level Examination Question from January 2012, IAL, FP1, Q8 (Edexcel)

$$\mathbf{A} = \begin{pmatrix} 0 & 1 \\ 2 & 3 \end{pmatrix}$$

(a) Show that **A** is non-singular

[2 marks]

(b) Find **B** such that $\mathbf{B}\,\mathbf{A}^2 = \mathbf{A}$

[4 marks]

Question 3

(i) Given that $\mathbf{AXBA} = \mathbf{I}$, prove that $\mathbf{X}^{-1} = \mathbf{BAA}$

[4 marks]

(ii) Given that $\mathbf{A} = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 2 & 1 \\ -3 & -1 \end{pmatrix}$, find $\mathbf{X}$

[4 marks]

Question 4

$$\mathbf{S} = \begin{pmatrix} 2 & 4 \\ 1 & 3 \end{pmatrix} \text{ and } \mathbf{B}^{-1} = \begin{pmatrix} 2 & 0 \\ 3 & 1 \end{pmatrix}$$

(a) Work out,

(i) $\mathbf{S}^{-1}$ **(ii)** $\mathbf{B}$

(iii) $\mathbf{BS}$ **(iv)** $(\mathbf{BS})^{-1}$

[8 marks]

(b) Verify the quotable result that if **B** and **S** are non-singular matrices then the LHS of $\mathbf{S}^{-1}\mathbf{B}^{-1} = (\mathbf{BS})^{-1}$ is equal to its RHS.

[3 marks]

(c) If **S** represents “putting on socks” and **B** represents “putting on boots” interpret what the result $(\mathbf{BS})^{-1} = \mathbf{S}^{-1}\mathbf{B}^{-1}$ represents.

[2 marks]

Question 5

Further A-Level Examination Question from May 2018, IAL, F1, Q4 (Edexcel)

$$\mathbf{A} = \begin{pmatrix} 2p & 3q \\ 3p & 5q \end{pmatrix} \qquad \text{where } p \text{ and } q \text{ are non-zero real constants}$$

(a) Find $\mathbf{A}^{-1}$ in terms of p and q

[3 marks]

Given $\mathbf{XA} = \mathbf{B}$, where $\mathbf{B} = \begin{pmatrix} p & q \\ 6p & 11q \\ 5p & 8q \end{pmatrix}$

(b) find the matrix **X**, giving your answer in its simplest form.

[4 marks]