Lesson 3

Further A-Level Pure Mathematics : Core 1 Matrix Transformations

3.1 The Multipoint Matrix

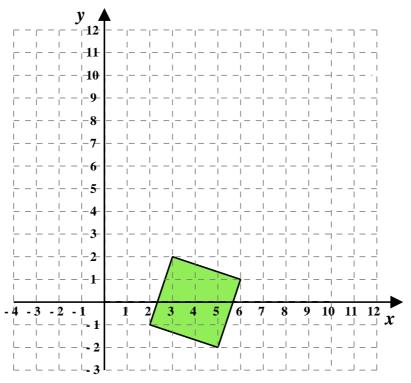
A square has vertices (3, 2), (6, 1), (5, -2) and (2, -1)

(i) Write the square's vertices as a multipoint matrix. and transform it with M

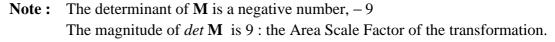
(ii) Transform the square's vertices using the matrix $\mathbf{M} = \begin{pmatrix} 1 & 4 \\ 2 & -1 \end{pmatrix}$

(**iii**) Add a plot of the transformed shape to the graph below. Teaching Video : <u>http://www.NumberWonder.co.uk/v9090/3.mp4</u>





[1, 4, 1 marks]



3.2 Exercise

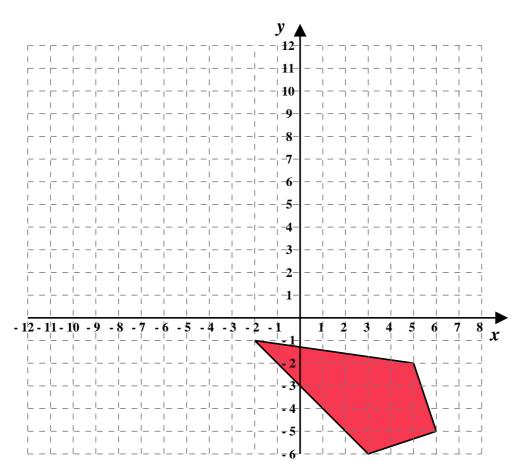
Any solution based entirely on graphical or numerical methods is not acceptable Marks Available : 30

Question 1

A kite has vertices (5, -2), (6, -5), (3, -6) and (-2, -1)

- (i) Write the kite's vertices as a multipoint matrix.
- (ii) Transform the kite's vertices using the matrix $\mathbf{M} = \begin{pmatrix} -2 & 0 \\ 0 & -2 \end{pmatrix}$
- (iii) Add a plot of the transformed shape to the graph below.

(iv) What is the area scale factor of the transformation ?

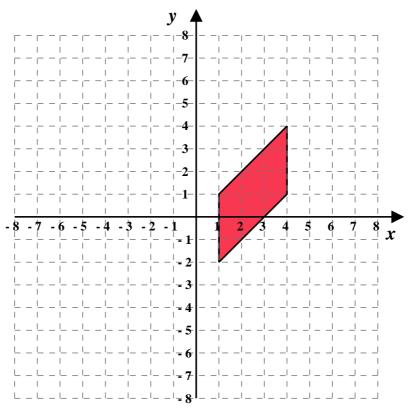


[1, 4, 1, 1 marks]

Question 2

A parallelogram has vertices (1, -2), (4, 1), (4, 4) and (1, 1)

- (i) Write the parallelogram's vertices as a multipoint matrix.
- (ii) Transform the parallelogram's vertices using the matrix $\mathbf{M} = \begin{pmatrix} 1 & -2 \\ -2 & 1 \end{pmatrix}$
- (iii) Add a plot of the transformed shape to the graph below.
- (iv) Find the magnitude of *det* M and explain what it tells you about the transformation.



[1,4,1,1 marks]

Question 3

This question is about working out the following matrix multiplication,

$$\left(\begin{array}{ccc} 5 & -1 & 6 \\ 8 & 3 & -4 \end{array}\right) \times \left(\begin{array}{ccc} 2 & 9 & 1 & -6 \\ -3 & 12 & -5 & 7 \\ 4 & -2 & 0 & 11 \end{array}\right)$$

Here is the multiplication grid already set up,

$$\begin{pmatrix} 2 & 9 & 1 & -6 \\ -3 & 12 & -5 & 7 \\ 4 & -2 & 0 & 11 \end{pmatrix}$$
$$\begin{pmatrix} 5 & -1 & 6 \\ 8 & 3 & -4 \end{pmatrix} \begin{pmatrix} 21 & & \\ & & -71 \end{pmatrix}$$

And here is how the 21 and the (-71) were found:

- In the product matrix, the 21 came from, $5 \times 9 + (-1) \times 12 + 6 \times (-2)$
- In the product matrix, the (-71) came from, $8 \times (-6) + 3 \times 7 + (-4) \times 11$

Complete the matrix multiplication grid.

[5 marks]

Question 4

Further A-Level Examination Question from January 2015, IAL, F1, Q6 (ii) (Edexcel)

$$\mathbf{M} = \begin{pmatrix} 2k + 5 & -4 \\ 1 & k \end{pmatrix} \text{ where } k \text{ is a real number}$$

Show that $det \mathbf{M} \neq 0$ for all values of k

Question 5

Further A-Level Examination Question from January 2014, IAL, F1, Q2 (Edexcel)

(i)
$$\mathbf{A} = \begin{pmatrix} -4 & 10 \\ -3 & k \end{pmatrix}$$
 where k is a constant.

The triangle T is transformed to the triangle T' by the transformation represented by **A**

Given that the area of triangle T' is twice the area of triangle T, find the possible values of k

[4 marks]

(**ii**) Given that,

 $\mathbf{B} = \begin{pmatrix} 1 & -2 & 3 \\ -2 & 5 & 1 \end{pmatrix} \qquad \mathbf{C} = \begin{pmatrix} 2 & 8 \\ 0 & 2 \\ 1 & -2 \end{pmatrix}$

find BC

[3 marks]

This document is a part of a **Mathematics Community Outreach Project** initiated by Shrewsbury School It may be freely duplicated and distributed, unaltered, for non-profit educational use In October 2020, Shrewsbury School was voted "**Independent School of the Year 2020**" © 2022 Number Wonder

Teachers may obtain detailed worked solutions to the exercises by email from mhh@shrewsbury.org.uk