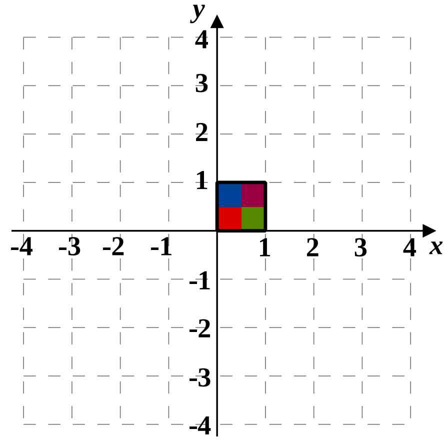
4.1 The Unit Square

Faced with an unfamiliar 2×2 transformation matrix, one way to investigate its properties is to apply it to a unit square.

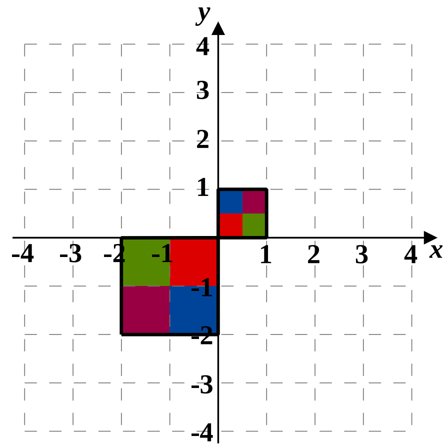
Written as a multipoint matrix the unit square to use is, $\mathbf{U} = \begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$

which is visualised as,



Consider the matrix, $\mathbf{M} = \begin{pmatrix} -2 & 0 \\ 0 & -2 \end{pmatrix}$ used in Exercise 3.2, Question 1.

MU	$\begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$
$\begin{pmatrix} -2 & 0 \\ 0 & -2 \end{pmatrix}$	$\begin{pmatrix} 0 & -2 & -2 & 0 \\ 0 & 0 & -2 & -2 \end{pmatrix}$



The transformation can be seen to be a rotation of 180° about the origin and an enlargement of Length Scale Factor 2.
The Area Scale Factor is 4, which is also given by *det* $\mathbf{M} = +\,4$.
The + sign indicates that the orientation is unchanged. In both the original shape and the image the colours go red, green, purple, blue in anticlockwise order.

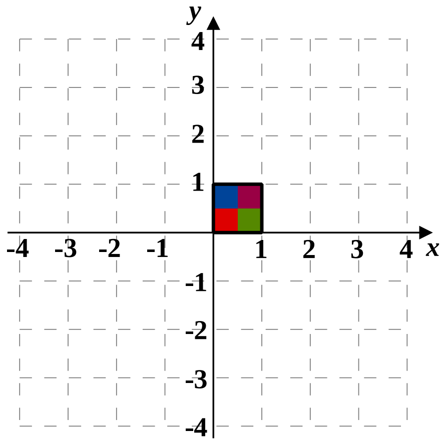
4.2 Exercise

Any solution based entirely on graphical
or numerical methods is not acceptable
Marks Available : 50

Question 1

A matrix, **N**, is used as a transformation, where $\mathbf{N} = \begin{pmatrix} -2 & -2 \\ -2 & 2 \end{pmatrix}$

(i) Apply **N** to the unit square and plot the resulting shape on the graph below.



[3 marks]

(ii) Calculate the determinant of **N**

[1 mark]

(iii) Explain carefully what the sign of the determinant tells you about the transformation, **N**

[2 marks]

(iv) What is the Length Scale Factor of the transformation ?

[1 mark]

(v) What is the Area Scale Factor of the transformation ?

[1 mark]

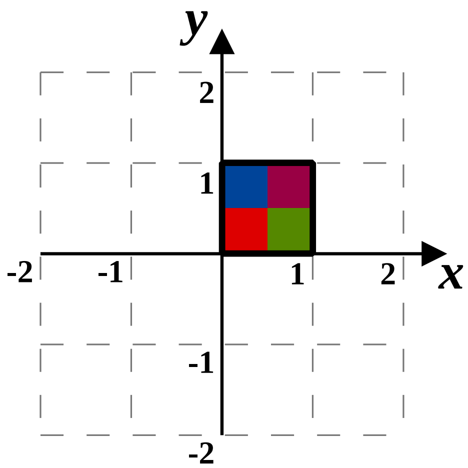
Question 2
Complete the following,
Ask your teacher to check your answers as it's important to get these correct !

A Catalogue of Two-Dimensional Transformations

$\begin{pmatrix}1&0\\0&1\end{pmatrix}$

	$\begin{pmatrix}0&1&1&0\\0&0&1&1\end{pmatrix}$
$\begin{pmatrix}1&0\\0&1\end{pmatrix}$	$\begin{pmatrix}0&&&\\0&&&\end{pmatrix}$

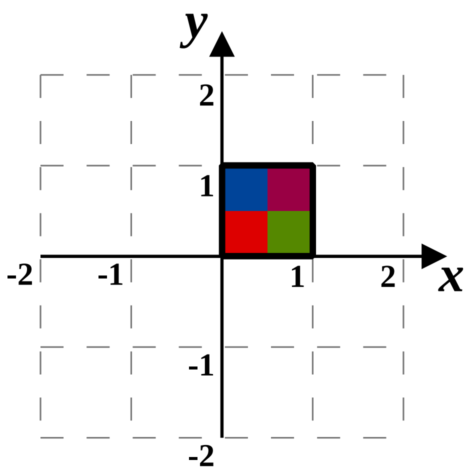
Description :



$\begin{pmatrix}0&-1\\1&0\end{pmatrix}$

	$\begin{pmatrix}0&1&1&0\\0&0&1&1\end{pmatrix}$
$\begin{pmatrix}0&-1\\1&0\end{pmatrix}$	$\begin{pmatrix}0&&&\\0&&&\end{pmatrix}$

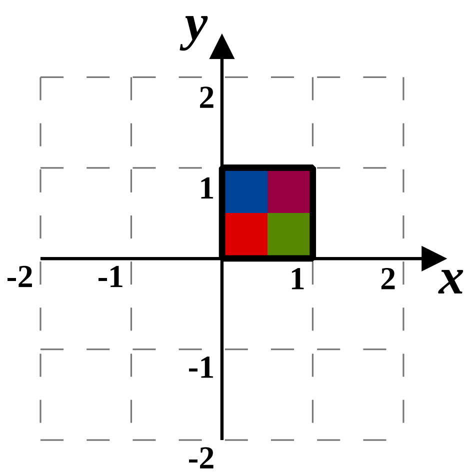
Description :



$\begin{pmatrix}-1&0\\0&-1\end{pmatrix}$

	$\begin{pmatrix}0&1&1&0\\0&0&1&1\end{pmatrix}$
$\begin{pmatrix}-1&0\\0&-1\end{pmatrix}$	$\begin{pmatrix}0&&&\\0&&&\end{pmatrix}$

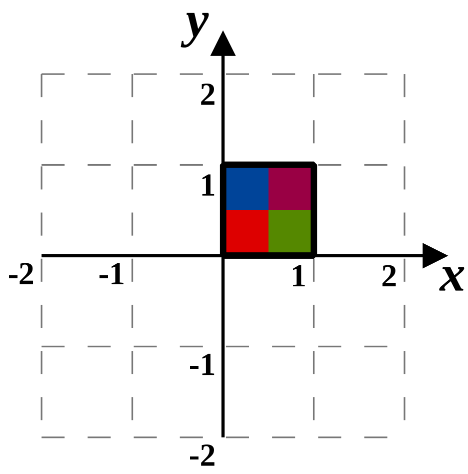
Description :



$$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

	$\begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$
$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & & & \\ 0 & & & \end{pmatrix}$

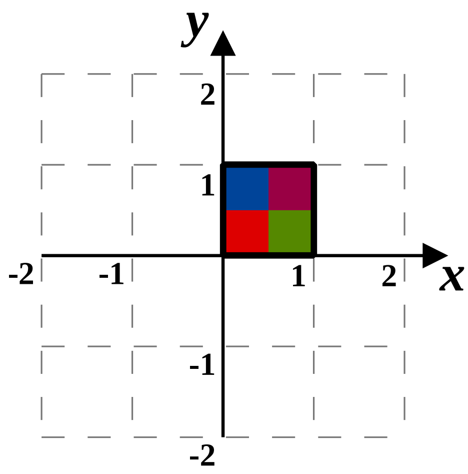
Description :



$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

	$\begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$
$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	$\begin{pmatrix} 0 & & & \\ 0 & & & \end{pmatrix}$

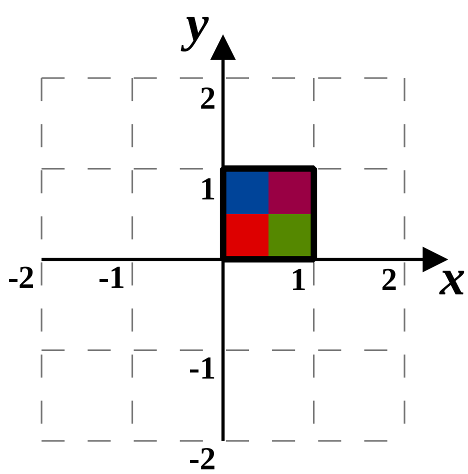
Description :



$$\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

	$\begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$
$\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 & & & \\ 0 & & & \end{pmatrix}$

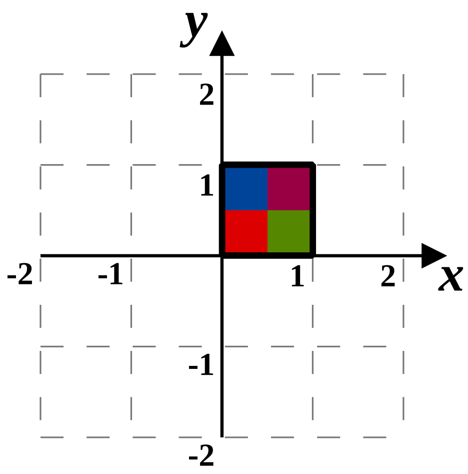
Description :



$$\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$$

	$\begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$
$\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & & & \\ 0 & & & \end{pmatrix}$

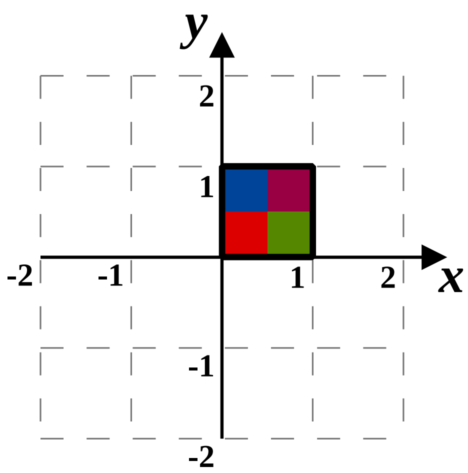
Description :



$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

	$\begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$
$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & & & \\ 0 & & & \end{pmatrix}$

Description :



[3 marks each = 24 marks total]

Question 3

Further A-Level Examination Question from May 2016, FP1, Q1 (Edexcel)

Given that k is a real number and that,

$$\mathbf{A} = \begin{pmatrix} 1+k & k \\ k & 1-k \end{pmatrix}$$

find the exact values of k for which $\mathbf{A}$ is a singular matrix.

Give your answers in their simplest form.

[3 marks]

Question 4

Further A-Level Examination Question from January 2011, FP1, Q2 (Edexcel)

$$\mathbf{A} = \begin{pmatrix} 2 & 0 \\ 5 & 3 \end{pmatrix}, \qquad \mathbf{B} = \begin{pmatrix} -3 & -1 \\ 5 & 2 \end{pmatrix}$$

(a) Find **AB**

[3 marks]

Given that,

$$\mathbf{C} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

(b) describe fully the geometrical transformation represented by **C**

[2 marks]

(c) write down $\mathbf{C}^{100}$

[1 mark]

Question 5

Further A-Level Examination Question from June 2011, FP1, Q3 (Edexcel)

(a) Given that $\mathbf{A} = \begin{pmatrix} 1 & \sqrt{2} \\ \sqrt{2} & -1 \end{pmatrix}$

(i) find $\mathbf{A}^2$

[3 marks]

(ii) describe fully the geometrical transformation represented by $\mathbf{A}^2$

[1 mark]

(b) Given that $\mathbf{B} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$

describe fully the geometrical transformation represented by **B**

[2 marks]

(c) Given that $\mathbf{C} = \begin{pmatrix} k+1 & 12 \\ k & 9 \end{pmatrix}$

where k is a constant, find the value of k for which the matrix **C** is singular

[3 marks]