GCSE Mathematics
Preparatory A-Level Mathematics

# ConiC SectionS

## ~ Simultaneous Equations IV ~

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**Lesson 1**

### 1.1 Perspectives

A typical question from a GCSE examination on simultaneous equations might ask candidates to solve, for example, this pair of equations,

$$y = 2x - 3$$
$$x^2 + y^2 = 2$$

In the rush to complete such a question under time pressure, most candidates will opt for a purely algebraic method. However, the majority of the mistakes made with such questions comes from not thinking from a perspective other than the algebraic.

Does “Time Pressure = No time to consider other points of view” ?

For the examination, being able to do the algebra is essential but outside of the examination hall a computer (called a symbolic manipulator) could do that for you; it's not the most important skill to learn from doing such questions.
What a computer can't do very well is see the problem from multiple perspectives.

Thinking geometrically, the upper of the two equations is a straight line; “obvious” because it is of the form $y = mx + c$.
The lower equation, as any A-Level mathematician will instantly tell you, is a circle.
They would tell you it is centred at the origin and has a radius of $\sqrt{2}$, about 1.414.

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**The Equation of a Circle**

$$x^2 + y^2 = r^2$$

This is a circle with centre ( 0, 0 ) and radius $r$

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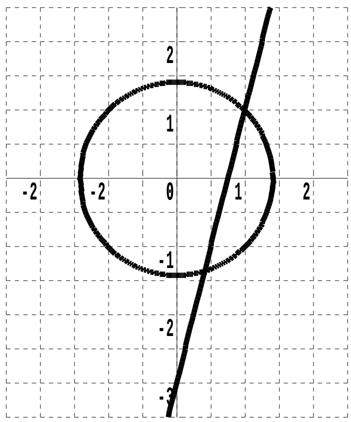
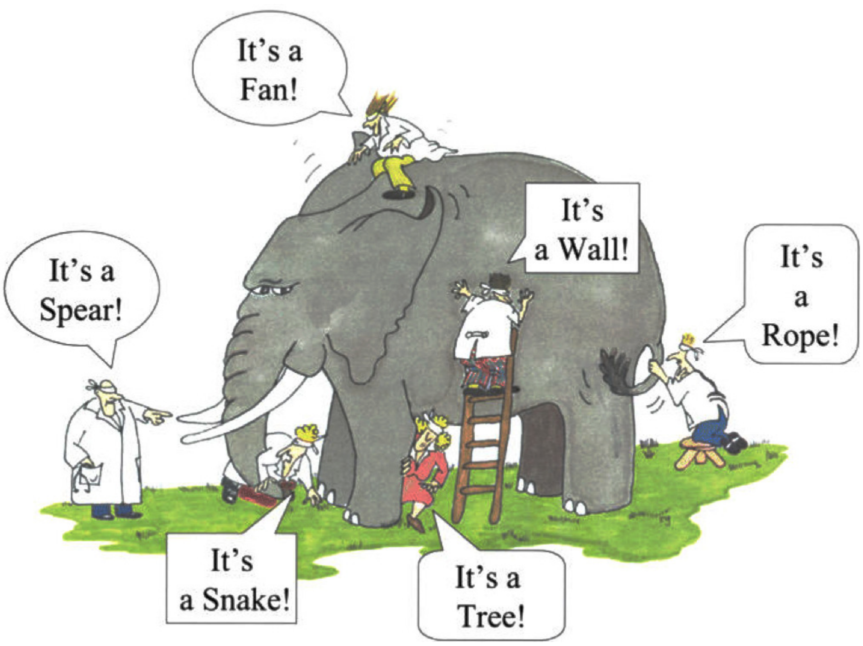
A knowledge of lines and circles allows us to sketch or plot the equations that are to be solved simultaneously. This is a new perspective on a algebraic problem.




The most obvious feature of the graph is that the line intersects the circle in two places. When solving these simultaneous equations the locations of those intersections are being found. So if a candidate gives only one answer and not two it reveals that they do not have a geometric perspective on the question !

Such intersections are points and a point has two parts, an *x* and a *y*. A common mistake is to find one part and forget to find the other and, again, this reveals a lack of a geometric perspectives of what the algebra is doing; that it's finding points.

From even a rough sketch, the approximate answers are apparent.
Many “obviously wrong” answers go unchecked in examinations because the candidate has not thought about viewing the algebra question from the different perspective of geometry.
In our question no $x$ or $y$ in the answer should be greater than $\sqrt{2}$ for example.




Being good at maths is being aware of different perspectives on the same problem

**1.2 The Algebraic Solution**

To solve the simultaneous equations

$$y = 2x - 3$$
$$x^2 + y^2 = 2$$

- Begin by using *the method of substitution.*

$$x^2 + y^2 = 2$$
$$x^2 + (2x - 3)^2 = 2$$

- Expand the brackets, FOIL

$$x^2 + (2x - 3)(2x - 3) = 2$$
$$x^2 + 4x^2 - 6x - 6x + 9 = 2$$

- Gather together like terms

$$5x^2 - 12x + 9 = 2$$

- Rearrange the equation into the form $f(x) = 0$

$$5x^2 - 12x + 7 = 0$$

- Factorise the quadratic

$$(5x - 7)(x - 1) = 0$$

- Apply The Product of Zero Theorem

$$\text{Either } 5x - 7 = 0 \quad \text{or} \quad x - 1 = 0$$
$$x = 1.4 \qquad\qquad x = 1$$

The answer is points where the line intersects the circle so we finally use the equation of the line $y = 2x - 3$ to work out $y$ when $x$ is 1.4, and 1.

The Final Answer

$$(1.4, -0.2) \text{ or } (1, -1)$$

- Don't forget to look back at the graph to check these are sensible answers.

Phew !

### 1.3 Exercise

**Question 1**

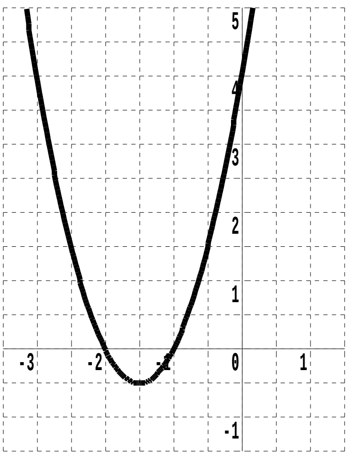
**( i )** Solve the equation

$$2x^2 + 6x + 4 = 0$$

by first dividing throughout by 2, then factorising, and then applying The Product of Zero Theorem. You should end up with two values of $x$ that make the original equation true.

**( ii )** Here is the graph of the equation,

$$y = 2x^2 + 6x + 4$$




Explain how you can check your part (i) answers are likely to be correct from this graph.

**Question 2**

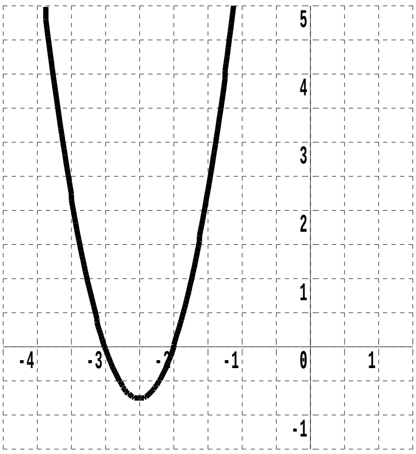
**( i )** Solve the equation

$$3x^2 + 15x + 18 = 0$$

by first dividing throughout by 3, then factorising, and then applying The Product of Zero Theorem. You should end up with two values of $x$ that make the original equation true.

**( ii )** Here is the graph of the equation,

$$y = 3x^2 + 15x + 18$$




Explain how you can check your part (i) answers are likely to be correct from this graph.

**Question 3**

**( i )** Solve the simultaneous equations

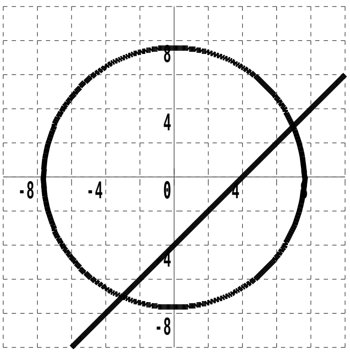
$$y = x - 4$$

$$x^2 + y^2 = 58$$

**( ii )** Here is the graph of the equations,

$y = x - 4$ (the line)

$x^2 + y^2 = 58$ (the circle)




Explain how you can check your part (i) answers are likely to be correct from this graph.

**Question 4**

*GCSE, June 2007, paper 3H, Q19*

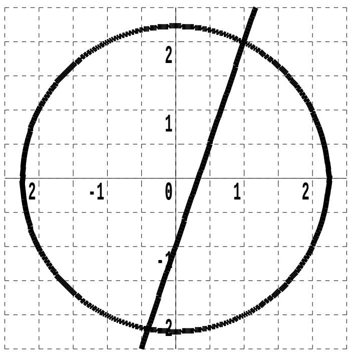
**( i )** Solve the simultaneous equations

$$y = 3x - 1$$
$$x^2 + y^2 = 5$$

**( ii )** Here is the graph of the equations,

$y = 3x - 1$ (the line)

$x^2 + y^2 = 5$ (the circle)




Explain how you can check your part (i) answers are likely to be correct from this graph.

**Question 5**

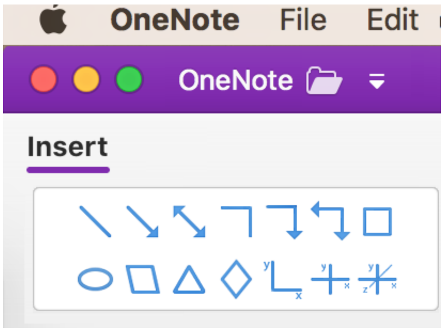
This is about using geometry (and NOT algebra) to solve the simultaneous equations,

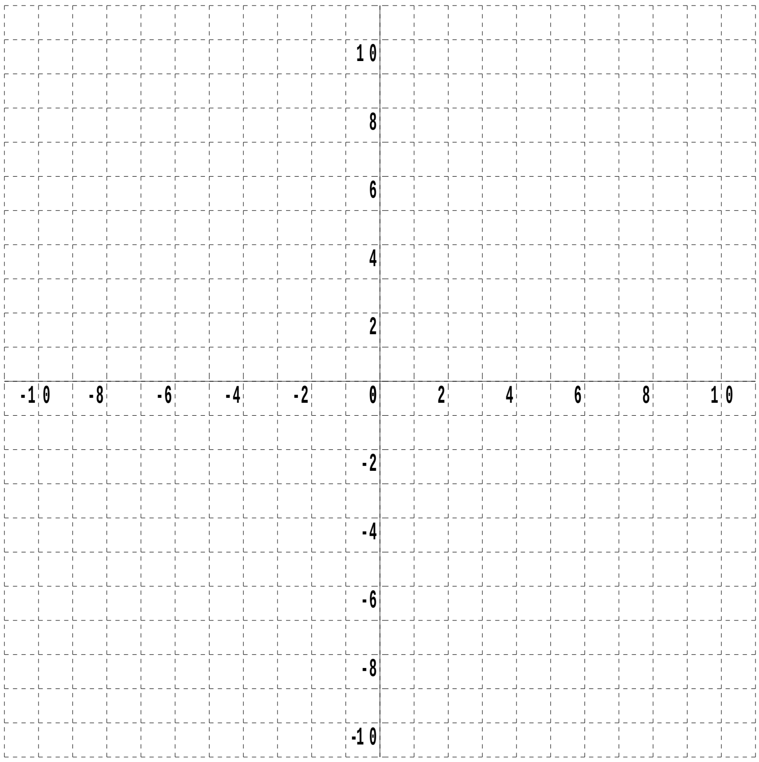
$$y = x - 7$$

$$x^2 + y^2 = 109$$

This is a valid method in this case because the points of intersection are integer and so a reasonably accurate plot will allow them to be read off the graph.

**( i )** Accurately plot the line and the circle on the graph.
On paper, use a compass to draw the circle.
On a computer, on OneNote for example, use the shapes tool.




**( ii )** From your graph, write down the solutions to the simultaneous equations.