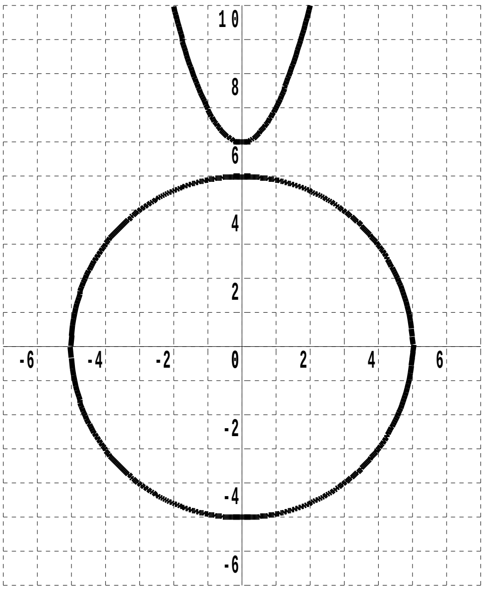
2.1 Circle and Parabola

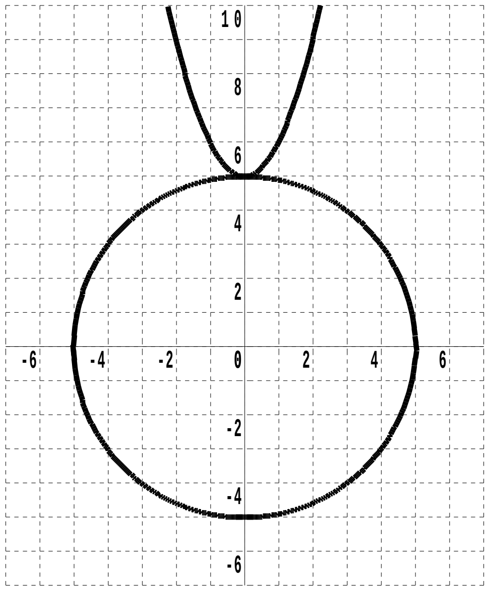
The circle with equation $x^2 + y^2 = 25$ has centre (0, 0) and radius 5 units.
Into this circle is going to be lowered a parabola, essentially $y = x^2$.
The lowering is achieved by varying the value of c in the equation $y = x^2 + c$.

An understanding of what might happen is gained from thinking geometrically.



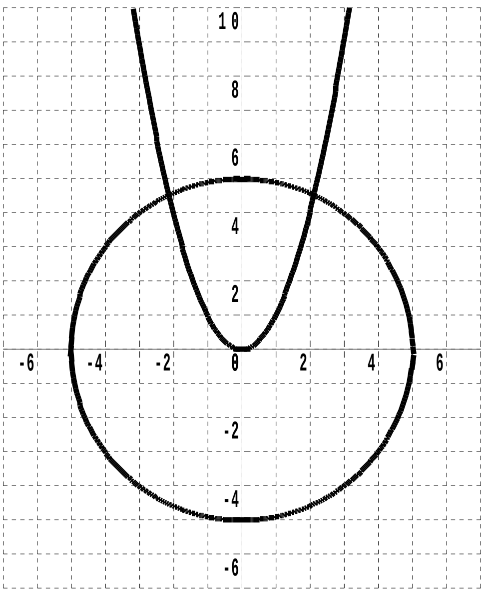
$x^2 + y^2 = 25$ with $y = x^2 + 6$

Zero solutions



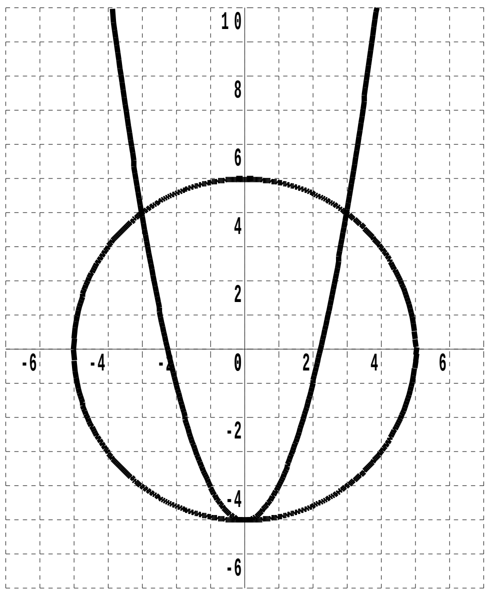
$x^2 + y^2 = 25$ with $y = x^2 + 5$

One Solution



$x^2 + y^2 = 25$ with $y = x^2$

Two solutions



$x^2 + y^2 = 25$ with $y = x^2 - 5$

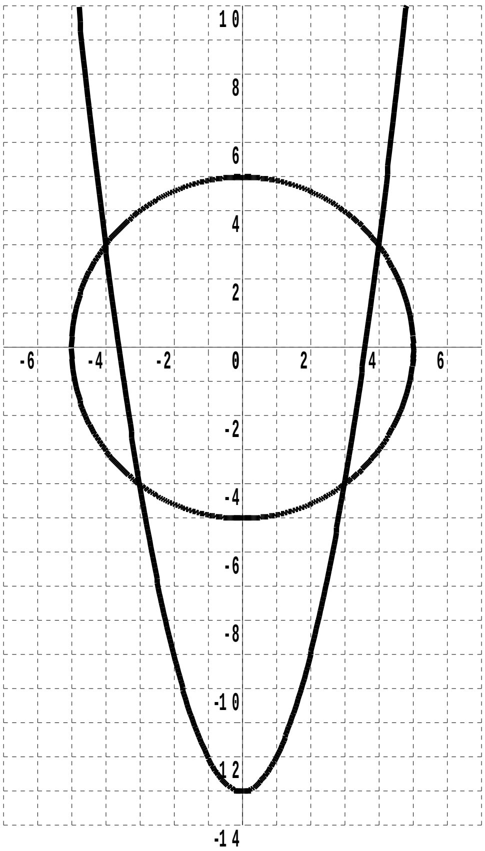
Three Solutions

The fifth and final possibility is that there are four solutions.

$$x^2 + y^2 = 25 \quad \text{with} \ y = x^2 - 13$$

For this four solution situation there is an obvious algebraic challenge; how will the algebra find so many solutions ?

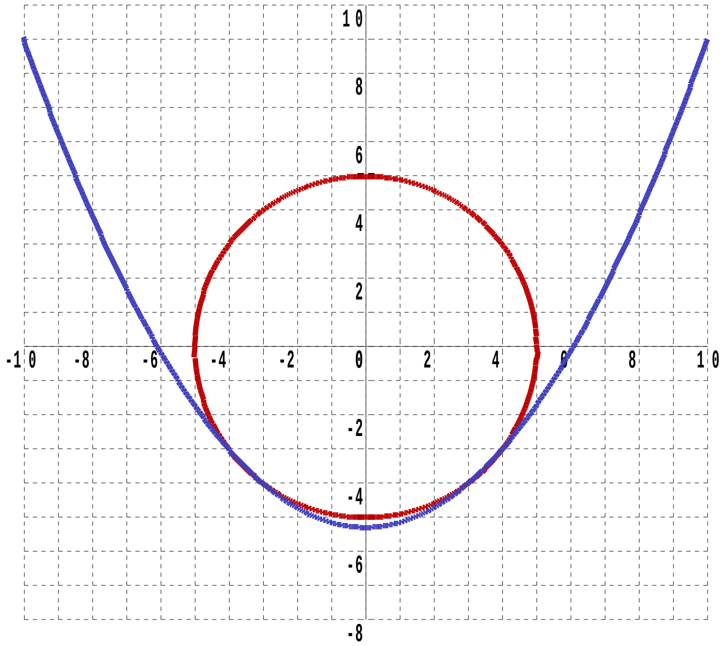
Teaching Videos : http://www.NumberWonder.co.uk/v9091/2.mp4



2.2 Exercise

Question 1

It's not always obvious from the graph how many points of intersection there are !

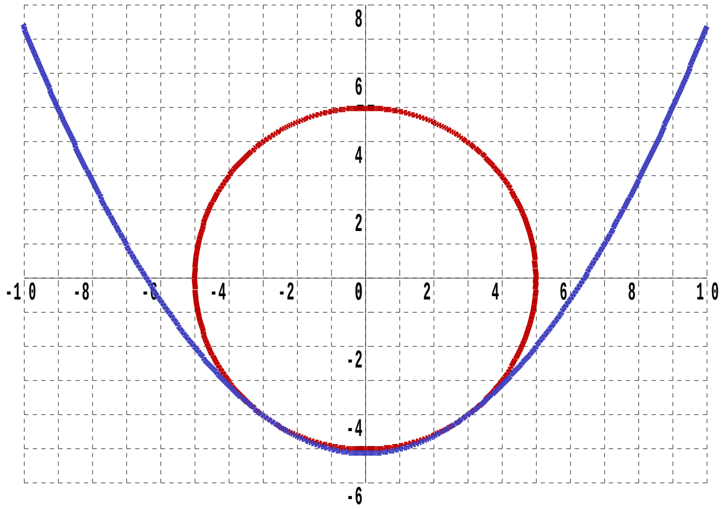


Use algebra to find all points of intersection of the circle $x^2 + y^2 = 25$

with the parabolic curve, $y = \dfrac{x^2 - 37}{7}$

Question 2

Peter has an idea that he can get many more than four points of intersection by running the quadratic curve along an arc of the circle. Here is his first attempt.

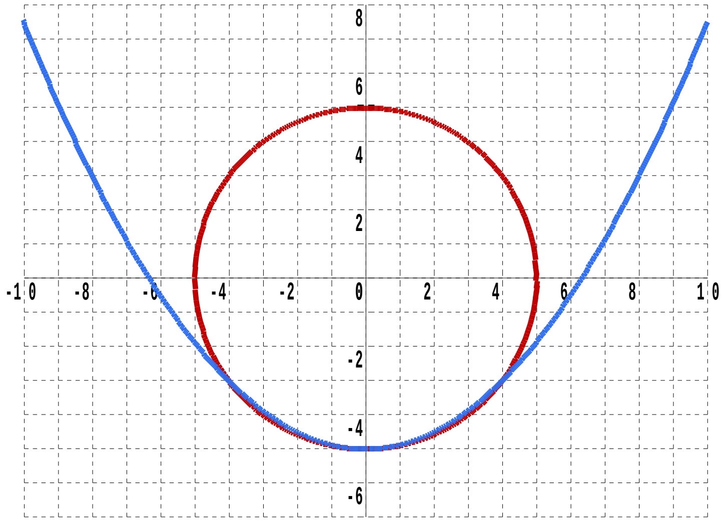


Use algebra to find all points of intersection of the circle $x^2 + y^2 = 25$

with the quadratic curve, $y = \dfrac{x^2 - 41}{8}$

Question 3

Peter is does not give up easily. Here is his second attempt.

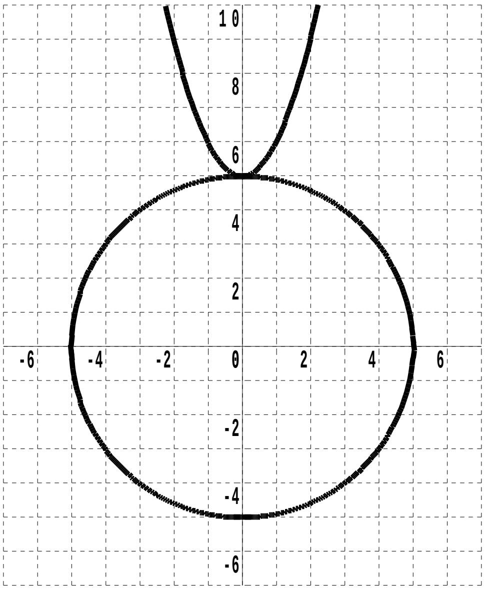


(i) Use algebra to find all points of intersection of the circle $x^2 + y^2 = 25$ with the quadratic curve, $y = \dfrac{x^2 - 40}{8}$

(ii) Do you think Peter's idea can be made to work ?
Yes or No ?

Question 4

In the introduction to this lesson, it was claimed that the circle $x^2 + y^2 = 25$ would only have one point of intersection with the parabola $y = x^2 + 5$



Verify this claim by using algebra to solve the two equations simultaneously.