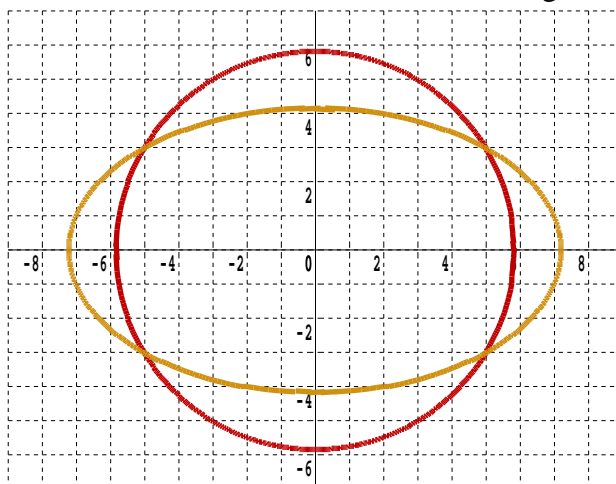## 4.1 Circle and Ellipse

In appearance, a circle (in red) and an ellipse (in gold) have much in common.
They are both smooth and continuous curves that form a graceful closed loop.

No surprise then that their algebraic equations also look similar.

$$\text{Red circle :} \qquad x^2 + y^2 = 34$$

$$\text{Gold ellipse :} \quad x^2 + 3\,y^2 = 52$$

More generally,

The Equation of a Circle

$$x^2 + y^2 = r^2$$

This is a circle with centre (0, 0) and radius r

The Equation of an Ellipse

$$x^2 + a\,y^2 = w^2$$

This is an ellipse with centre (0, 0) and *half width, w*

In fact, when $a = 1$ in the equation of an ellipse, it becomes the equation of a circle.
This is because,

$$a = \left(\frac{\textit{half width}}{\textit{half height}} \right)^2$$

and for a circle the *half width* and the *half height* are the same; the *radius*, *r*.
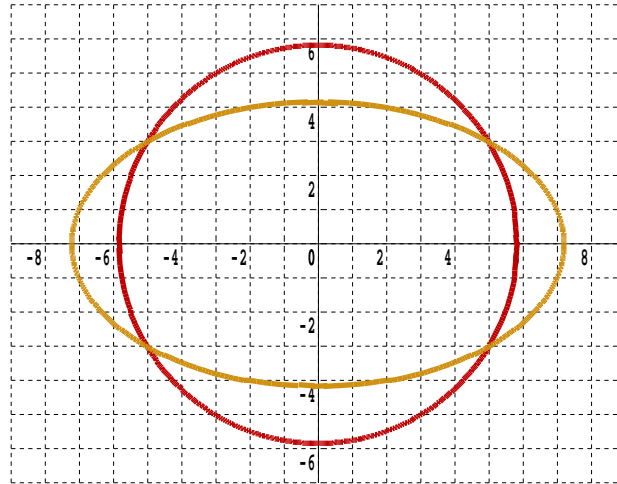- The *half width* is the number where the ellipse crosses the positive *x*-axis.
- The *half height* is the number where the ellipse crosses the positive *y*-axis.

4.2 Where Gold meets Red

Use algebra to solve the simultaneous equations,

$$\text{Red circle}: \quad x^2 + y^2 = 34$$

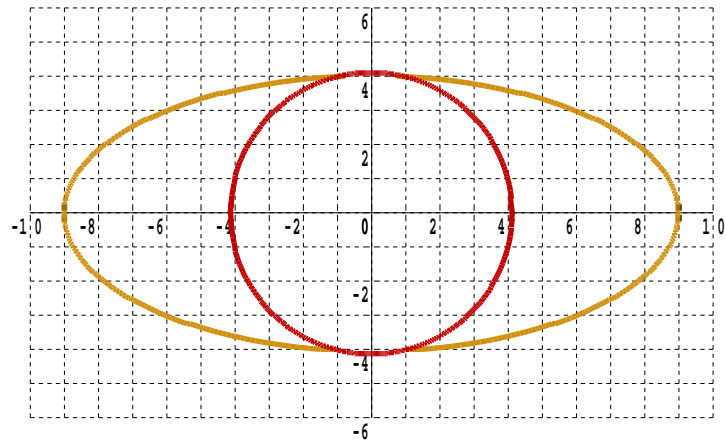$$\text{Gold ellipse}: \quad x^2 + 3y^2 = 52$$

Teaching Video : http://www.NumberWonder.co.uk/v9091/4.mp4

4.3 Exercise

Question 1

Use algebra to solve the simultaneous equations,

$$\text{Red circle}: \quad x^2 + y^2 = 17$$

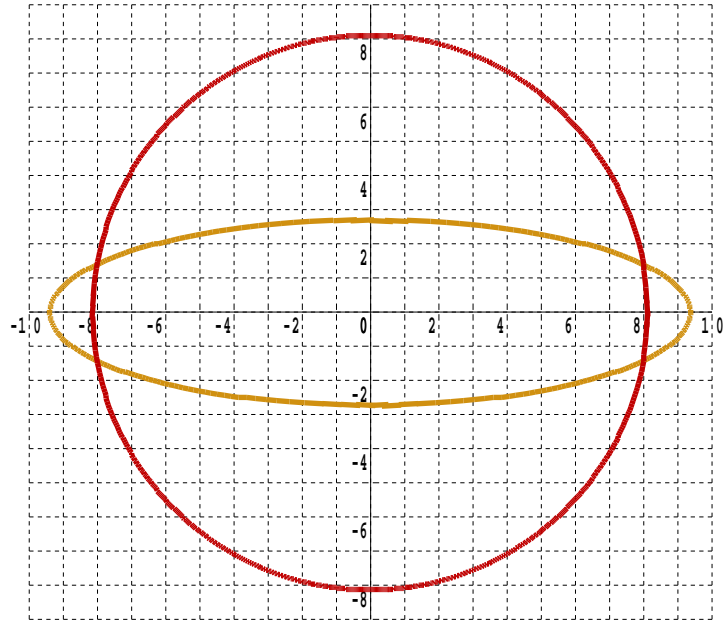$$\text{Gold ellipse}: \quad x^2 + 5\,y^2 = 81$$

Question 2

Use algebra to solve the simultaneous equations,

$$\text{Red circle}: \quad x^2 + \quad y^2 \ = \ 66$$

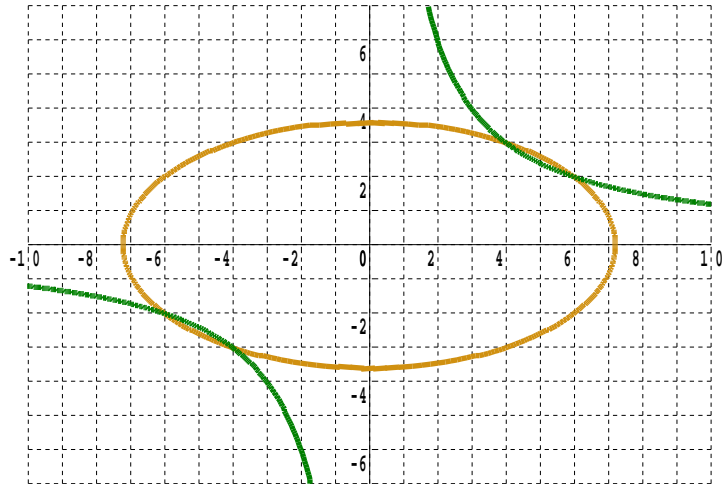$$\text{Gold ellipse}: \quad x^2 + 12\, y^2 \ = \ 88$$

Give your points as exact coordinates, leaving square roots in your answers.

Question 3

Use algebra to solve the simultaneous equations,

$$\text{Gold ellipse :} \quad x^2 + 4y^2 = 52$$

$$\text{Green hyperbola :} \quad y = \frac{12}{x}$$

You may find this list of pairs of positive integers that have a product of 576 of use !

1×576

2×288

3×192

4×144

6×96

8×72

9×64

12×48

16×36

18×32

24×24

Question 4

Use algebra to solve the simultaneous equations,

$$\text{Gold ellipse} : \quad x^2 + 2\,y^2 \;=\; 72$$

$$\text{Purple parabola} : \qquad y \;=\; \frac{x^2}{8} - 6$$