Lesson 4

Further A-Level Pure Mathematics
Vectors III : Core 1

4.1 Shortest Distance from Point to Line

Example

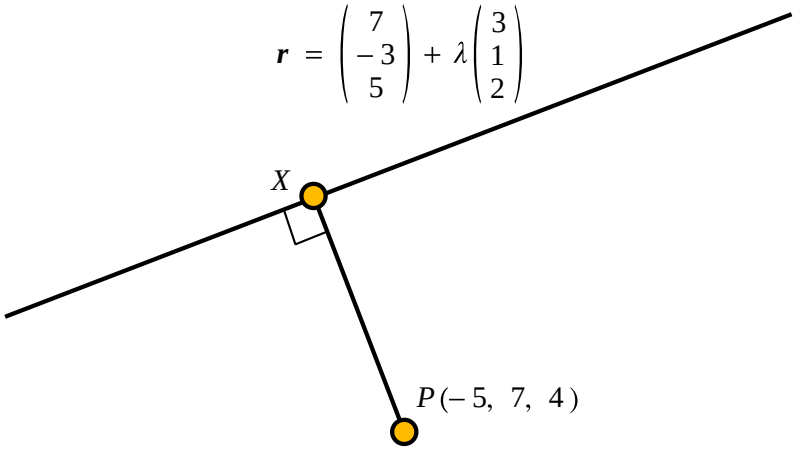
Question :

Find the shortest distance between the point $P(-5, 7, 4)$

and the line with equation $\boldsymbol{r} = \begin{pmatrix} 7 \\ -3 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix}$

Answer :

Call the location on the line that is closest to P the point X



[4 marks]

4.2 Exercise

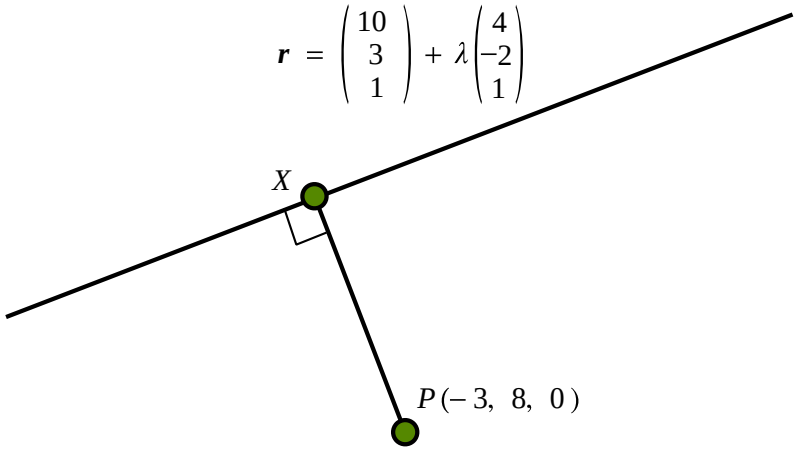
Any solution based entirely on graphical
or numerical methods is not acceptable
Marks Available : 50

Question 1

Find the shortest distance between the point $P(-3, 8, 0)$ and the line with equation

$$\boldsymbol{r} = \begin{pmatrix} 10 \\ 3 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} 4 \\ -2 \\ 1 \end{pmatrix}$$

Begin by calling the location on the line that is closest to P the point X



[4 marks]

Question 2

Find the shortest distance between the point $P(7, -1, -6)$ and the line with equation

$$\boldsymbol{r} = \begin{pmatrix} -7 \\ 15 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ -4 \\ 1 \end{pmatrix}$$

[5 marks]

Question 3

C4 Examination Question from January 2011, Q4

Relative to a fixed origin O, the point A has position vector $\boldsymbol{i} - 3\boldsymbol{j} + 2\boldsymbol{k}$ and the point B has position vector $-2\boldsymbol{i} + 2\boldsymbol{j} - \mathbf{k}$

The points A and B lie on a straight line l.

(a) Find $\overrightarrow{AB}$

[2 marks]

(b) Find a vector equation of l

[2 marks]

The point C has position vector $2\boldsymbol{i} + p\boldsymbol{j} - 4\boldsymbol{k}$ with respect to O, where p is a constant.

Given that AC is perpendicular to l, find

(c) the value of p

[4 marks]

(d) the distance AC

[2 marks]

Question 4

With respect to a fixed origin O, the lines l_1 and l_2 are given by the equations

$$l_1 \ : \ \boldsymbol{r} = (9\boldsymbol{i} + 13\boldsymbol{j} - 3\boldsymbol{k}) + \lambda(\boldsymbol{i} + 4\boldsymbol{j} - 2\boldsymbol{k})$$

$$l_2 \ : \ \boldsymbol{r} = (2\boldsymbol{i} - \boldsymbol{j} + \boldsymbol{k}) + \mu(2\boldsymbol{i} + \boldsymbol{j} + \boldsymbol{k})$$

where λ and μ are scalar parameters.

(a) Given that l_1 and l_2 meet, find the position vector of their point of intersection.

[5 marks]

(b) Find the acute angle between l_1 and l_2, giving your answer in degrees to 1 decimal place

[3 marks]

Given that the point A has position vector $4\boldsymbol{i} + 16\boldsymbol{j} - 3\boldsymbol{k}$ and that the point P lies on l_1 such that AP is perpendicular to l_1,

(c) Find the exact coordinates of P

[6 marks]

Question 5

P3 Examination Question from January 2002, Q6

Relative to a fixed origin O, the point A has position vector $4\,\boldsymbol{i} + 8\,\boldsymbol{j} - \boldsymbol{k}$, and the point B has position vector $7\,\boldsymbol{i} + 14\,\boldsymbol{j} + 5\,\boldsymbol{k}$

(a) Find the vector $\overrightarrow{AB}$

[1 mark]

(b) Calculate the cosine of $\angle$OAB

[3 marks]

(c) Show that, for all values of λ, the point P with position vector $\lambda\,\boldsymbol{i} + 2\lambda\,\boldsymbol{j} + (2\lambda - 9)\,\boldsymbol{k}$ lies on the line through A and B

[3 marks]

(d) Find the value of λ for which OP is perpendicular to AB

[3 marks]

(e) Hence find the coordinates of the foot of the perpendicular from O to AB

[2 marks]