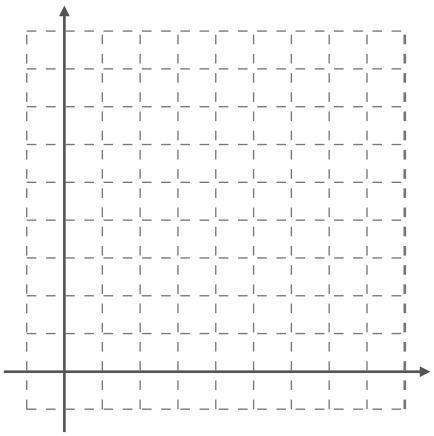
Lesson 5

Further A-Level Pure Mathematics
Vectors III : Core 1

5.1 Which Angle From The Scalar Product ?

$\triangle ABC$ is formed from the points A (2, 3), B(5, 1) and C (4, 7)

Determine the size of $\angle$CAB in degrees correct to one decimal place.



[3 marks]

5.2 Exercise

Any solution based entirely on graphical or numerical methods is not acceptable

Marks Available : 50

Question 1

You are given that,

$$\overrightarrow{AB} = \begin{pmatrix} -4 \\ 1 \\ -8 \end{pmatrix} \quad \text{and} \quad \overrightarrow{BC} = \begin{pmatrix} 3 \\ -2 \\ -6 \end{pmatrix}$$

(i) Write down the vector $\overrightarrow{BA}$

[1 mark]

(ii) Determine $\left|\overrightarrow{BA}\right|$

[1 mark]

(iii) Determine $\left|\overrightarrow{BC}\right|$

[1 mark]

(iv) Determine $\angle ABC$ in degrees, accurate to 1 decimal place.

[2 marks]

(v) Find the area of $\triangle ABC$

[1 mark]

Question 2

$\triangle ABC$ is formed from the points $A(-4, 7, 3)$, $B(-8, 5, 2)$ and $C(9, 1, -6)$

Determine the size of $\angle$BAC in degrees correct to one decimal place.

[3 marks]

Question 3

C4 Examination Question from January 2007, Q7

The point A has position vector $\boldsymbol{a} = 2\boldsymbol{i} + 2\boldsymbol{j} + \boldsymbol{k}$ and the point B has position vector $\boldsymbol{b} = \boldsymbol{i} + \boldsymbol{j} - 4\boldsymbol{k}$ relative to an origin O.

(a) Find the position vector of the point C, with position vector $\boldsymbol{c}$ given by $\boldsymbol{c} = \boldsymbol{a} + \boldsymbol{b}$

[1 mark]

(b) Show that $OACB$ is a rectangle, and find its exact area

[6 marks]

The diagonals of the rectangle, AB and OC meet at the point D

(c) Write down the position vector of the point D

[1 mark]

(d) Find the size of the angle ADC

[6 marks]

Question 4

C4 Examination Question from June 2010, Q7 (edited)

The line l_1 has equation;

$$\boldsymbol{r} = \begin{pmatrix} 2 \\ 3 \\ -4 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$$

where λ is a scalar parameter.

The line l_2 has equation;

$$\boldsymbol{r} = \begin{pmatrix} 0 \\ 9 \\ -3 \end{pmatrix} + \mu \begin{pmatrix} 5 \\ 0 \\ 2 \end{pmatrix}$$

where μ is a scalar parameter.

Given that l_1 and l_2 meet at the point C, find

(a) the coordinates of C

[3 marks]

The point A is the point on l_1 where $\lambda = 0$ and the point B is the point on l_2 where $\mu = -1$

(b) Find the size of the angle ACB.
Give your answer in degrees to 2 decimal places.

[4 marks]

(c) Hence, or otherwise, find the area of the triangle ABC

[5 marks]

Question 5

C4 Examination Question from January 2012, Q7

Relative to a fixed origin O, the point A has position vector $2\boldsymbol{i} - \boldsymbol{j} + 5\boldsymbol{k}$,
the point B has position vector $5\boldsymbol{i} + 2\boldsymbol{j} + 10\boldsymbol{k}$
and the point D has position vector $-\boldsymbol{i} + \boldsymbol{j} + 4\boldsymbol{k}$

The line l passes through the points A and B

(a) Find the vector $\overrightarrow{AB}$

[2 marks]

(b) Find a vector equation for the line l

[2 marks]

(c) Show that the size of the angle BAD is 109°, to the nearest degree.

[4 marks]

The points *A*, *B* and *D*, together with a point *C*, are the vertices of the parallelogram *ABCD*, where $\overrightarrow{AB} = \overrightarrow{DC}$

(d) Find the position vector of *C*

[2 marks]

(e) Find the area of the parallelogram *ABCD*, giving your answer to 3 significant figures.

[3 marks]

(f) Find the shortest distance from the point *D* to the line *l*, giving your answer to 3 significant figures.

[2 marks]