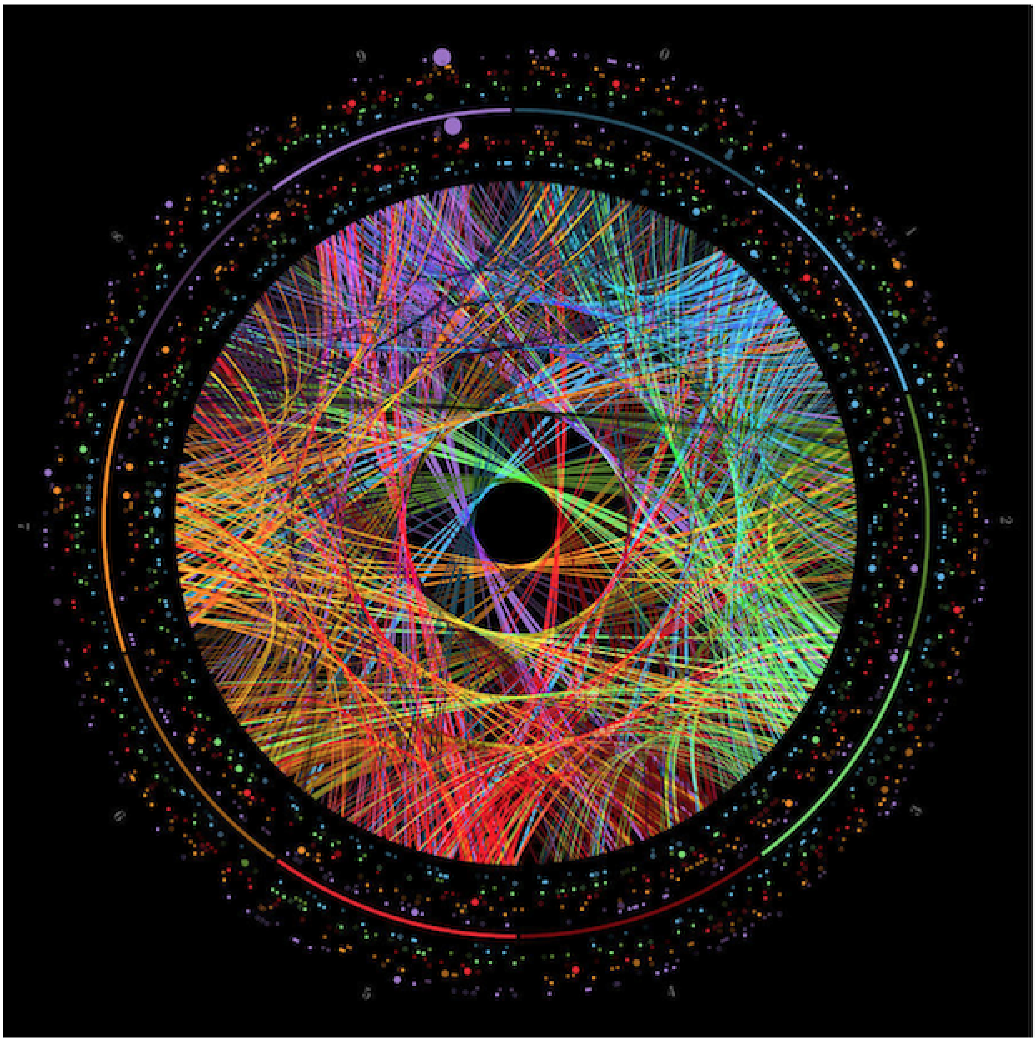
Further Pure A-Level Mathematics
Compulsory Course Component
Core 1

MATRIX SYSTEMS OF EQUATIONS



“Pi Transition Paths” by Martin Krzywinski

MATRIX
SYSTEMS OF EQUATIONS

Lesson 1

1.1 Matrix as Organiser

A matrix can be viewed as an organiser; a tool that allows information to be processed in a systematic, compact and efficient manner. When solving, for example, a system of five simultaneous equations with five unknowns, using matrices keeps the working both manageable and presentable. Computerising the solution techniques developed is then straight forward as most computer languages have matrix arithmetic built in.

1.2 Two Equations, Two Unknowns

As a simple example consider the following two simultaneous equations with two unknown variables x and y,

$$2x + 5y = 4$$

$$6x + 4y = 1$$

These are rewritten as the following matrix calculation,

$$\begin{pmatrix} 2 & 5 \\ 6 & 4 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$$

The theory for this rewrite in matrix form was covered in Lesson 2 of *Matrix Transformations* where how to transform a point (x, y) using a matrix was studied.
Here is a reminder of that key previous result;

Point Transformation by a Matrix (Two Dimensions)

The point (x, y) is written $\begin{pmatrix} x \\ y \end{pmatrix}$ and placed right of the transforming matrix.

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} ax + by \\ cx + dy \end{pmatrix}$$

The point (x, y) has been transformed to the point $(ax + by,\ cx + dy)$

From GCSE it is known that the algebra of solving two simultaneous equations has a geometric interpretation, namely, of finding the point where two straight lines intersect. So matrices, previously used to manipulate points, are also of use in solving simultaneous equations because solving them is all about finding a point; the point (x, y) of intersection of the two straight lines.

To advance the example, another previous result is needed, that of writing down the inverse matrix of a given matrix,

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The Inverse of a Matrix

In general, the inverse of a non-singular matrix $\mathbf{M}$ is the matrix $\mathbf{M}^{-1}$ such that

$$\mathbf{M}\mathbf{M}^{-1} = \mathbf{M}^{-1}\mathbf{M} = \mathbf{I}$$

In particular, if $\mathbf{M} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, then $\mathbf{M}^{-1} = \dfrac{1}{det\,\mathbf{M}} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

This result in turn required knowing how to find the determinant of a matrix,

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Definition of the Determinant

Given the generalised 2×2 square matrix, $\mathbf{M}$, where,

$$\mathbf{M} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

the determinant $\mathbf{M}$ is given by,

$$|\mathbf{M}| = ad - bc$$

This can also be written $det\,\mathbf{M}$ or $\Delta\mathbf{M}$

- If $|\mathbf{M}| = 0$ then $\mathbf{M}$ is a singular matrix
- If $|\mathbf{M}| \neq 0$ then $\mathbf{M}$ is a non-singular matrix

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At last, armed with this background knowledge, the example can be progressed !
Teaching Video : http://www.NumberWonder.co.uk/v9095/1.mp4

Watch the video and then write out a full solution here:

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[5 marks]

1.3 Exercise

Any solution based entirely on graphical
or numerical methods is not acceptable
Marks Available : 30

Question 1

Let **P** be the matrix $\begin{pmatrix} 7 & 5 \\ 3 & 2 \end{pmatrix}$ and let **Q** be the matrix $\begin{pmatrix} 6 \\ 1 \end{pmatrix}$

Calculate the matrix **PQ**

[3 marks]

Question 2

In the teaching video, it was claimed that when the Left Hand Side of

$$\begin{pmatrix} 2 & 5 \\ 6 & 4 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$$

was front multiplied by $\mathbf{M}^{-1}$ the result was just $\begin{pmatrix} x \\ y \end{pmatrix}$

Show in more detail exactly how this comes about.

[4 marks]

Question 3

Let **A** be the matrix $\begin{pmatrix} 10 & 4 \\ 7 & 3 \end{pmatrix}$

Determine $\mathbf{A}^{-1}$

[4 marks]

Question 4

Let **M** be the matrix $\begin{pmatrix} 0.5 & 0.25 \\ 2 & 1.5 \end{pmatrix}$

(i) Calculate the value of $det(\mathbf{M})$

[2 marks]

(ii) Determine $\mathbf{M}^{-1}$

Simplify your answer

[2 marks]

Question 5

Solve the following simultaneous equations using matrix methods,

$$\begin{aligned} 3x + 4y &= 18 \\ 4x - y &= 5 \end{aligned}$$

[5 marks]

Question 6

Solve the following simultaneous equations using matrix methods,

$$5x - 2y = 16$$

$$7x + 6y = -4$$

[5 marks]

Question 7

The matrix $\mathbf{A} = \begin{pmatrix} 1 & -3 \\ 2 & 1 \end{pmatrix}$ and $\mathbf{AB} = \begin{pmatrix} 4 & 1 & 9 \\ 1 & 9 & 4 \end{pmatrix}$

Find the matrix **B**

[5 marks]