Lesson 4

4.1 Three Equations, Three Unknowns

The previous two lessons have developed the mathematics necessary to solve a set of three simultaneous equations in three unknowns using matrix methods.

The strategy employed is exactly the same as that used in Lesson 1 when questions of two equations in two unknowns where tackled.

Example

Use your calculator help find the unique solution to the system of equations,

$$\begin{aligned} 2x + 4y - z &= 12 \\ x - y + 4z &= 6 \\ 4x + 5y - z &= 17 \end{aligned}$$

Teaching Instructions :

How to use a CASIO fx-991EX to help solve this is presented on the next page.

[4 marks]

Calculator Assisted Solution using the CASIO CLASSWIZ fx-991EX

First write the system of equations as a matrix equation,

$$\begin{pmatrix} 2 & 4 & -1 \\ 1 & -1 & 4 \\ 4 & 5 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 12 \\ 6 \\ 17 \end{pmatrix}$$

In what follows $\mathbf{A} = \begin{pmatrix} 2 & 4 & -1 \\ 1 & -1 & 4 \\ 4 & 5 & -1 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 12 \\ 6 \\ 17 \end{pmatrix}$

Use the calculator to get the inverse of matrix **A** as follows,

- Turn the calculator **ON** and **MENU 4** to get into matrix mode .
- Press **1** to define matrix **A**
- Press **3** and **3** again to specify 3 rows and 3 columns for matrix **A**
- Enter the nine elements of the matrix **A** pressing **=** after each entry
- Press **AC** to tell the calculator the matrix **A** is now defined
- Press **OPTN 3** to initiate a calculation involving matrix **A**
- Press the button $\boldsymbol{x^{-1}}$ followed by **=**
- Scroll through the elements of the inverse matrix and write down,

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{21}\begin{pmatrix} -19 & -1 & 15 \\ 17 & 2 & -9 \\ 9 & 6 & -6 \end{pmatrix}\begin{pmatrix} 12 \\ 6 \\ 17 \end{pmatrix}$$

The above line of working is worth half marks.

Now the calculator will be used to perform the matrix multiplication $\mathbf{A}^{-1}\mathbf{B}$ and so yield the values of x, y and z

- Press **MENU 4** to again enter the Define Matrix screen
- Press **2** to define matrix **B**
- Press **3** and **1** to specify 3 rows and 1 column for matrix **B**
- Enter the three elements of the matrix **B** pressing **=** after each entry
- Press **AC** to tell the calculator the matrix **B** is now defined
- Press **OPTN 3** to initiate a calculation involving matrix **A**
- Press the button $\boldsymbol{x^{-1}}$ followed by $\times$
- Press **OPTN 4** to enter matrix **B** into the evolving calculation
- Now press **=** and write down,

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}$$

$\therefore$ The unique solution is $x = 1,\ y = 3,\ z = 2$

[4 marks]

4.2 Exercise

Any solution based entirely on graphical
or numerical methods is not acceptable

Marks Available : 30

Question 1

Further A-Level Examination Question, May 2018, Core 1, Q1 (a), (b) (Edexcel)

$$\mathbf{M} = \begin{pmatrix} 2 & 1 & -3 \\ 4 & -2 & 1 \\ 3 & 5 & -2 \end{pmatrix}$$

(a) Find $\mathbf{M}^{-1}$ giving each element in exact form.

[2 marks]

(b) Solve the simultaneous equations,

$$2x + y - 3z = -4$$
$$4x - 2y + z = 9$$
$$3x + 5y - 2z = 5$$

[2 marks]

Question 2

In the following system of equations, a is an unknown constant, $a \neq -2$,

$$\begin{aligned} x - y + z &= 4 \\ 4x + z &= 2a \\ 2x + ay + 2z &= a \end{aligned}$$

(i) Construct a suitable matrix equation with a view to preparing to solve this system of equations by matrix methods.

[1 mark]

(ii) Find, in terms of a, an expression for the determinant of the matrix,

$$\mathbf{S} = \begin{pmatrix} 1 & -1 & 1 \\ 4 & 0 & 1 \\ 2 & a & 2 \end{pmatrix}$$

[2 marks]

(iii) From **S**, form the matrix of minors, **M**, in terms of a

[4 marks]

(iv) From **M**, form the matrix of cofactors, **C**, in terms of a

[1 mark]

(v) Write down the transpose, $\mathbf{C}^{\mathrm{T}}$, of the matrix of cofactors, in terms of a

[1 mark]

(vi) Write down in terms of a the inverse matrix $\mathbf{S}^{-1}$

[1 mark]

(vii) Find, in terms of a, the values of x, y, and z

[2 marks]

(viii) Show that if $a = 3$, the values of x, y and z are integers.

[2 marks]

Question 3

Three planes A, B and C are defined by the following equations;

$$A : \quad x + y + z = 3$$

$$B : \quad 2x - y - z = 0$$

$$C : \quad 3x - 2y + z = -1$$

By constructing and solving a suitable matrix equation, show that these three planes intersect at a single point and find the coordinates of that point

[5 marks]

Question 4

Use your calculator to find the inverse of,

$$\begin{pmatrix} 1 & 1 & 0 & 1 \\ 2 & 3 & 1 & 4 \\ 0 & 1 & 2 & 2 \\ 0 & 2 & 3 & 5 \end{pmatrix}$$

[4 marks]

Question 5

Find the inverse of the matrix $\begin{pmatrix} 1 & 0 & 0 \\ 0 & k & 0 \\ 0 & 0 & 1 \end{pmatrix}$ where k is a constant

[3 marks]