Lesson 3

Further A-Level Pure Mathematics, Core 2

Polar Coordinates

3.1 Old Friends in New Clothes

To properly understand a polar curve, there is no substitute for going to the effort of plotting it by hand. Thought should be applied to focussing on any parts of the curve where it's less predictable, and on any transitions where *r* flips sign, which can be the result of $r \to \pm\infty$ or $r \to 0$.

In many situations it will be desirable to insist that *r* be non-negative. However, on looking back at the polar plot of the first lesson, excluding the short piece of curve where *r* became negative would have destroyed the symmetry and flow of the curve. Many text books, internet articles and videos on polar curves are not clear if what is being discussed applies only for $r \geqslant 0$, or for all real values of *r*. This is a source of muddle and confusion as is often makes no difference but sometimes does !

3.2 Exercise

Marks Available : 50

Question 1

Here is a polar equation which is to be graphed.

$$r(\theta) = \frac{8}{2\cos\theta + \sin\theta}$$

(i) Explain why $r(90)$ and $r(270)$ yield the same point on the graph

[2 marks]

(ii) Complete the following table.
Notice the table is "intelligent" with a focus on key features.

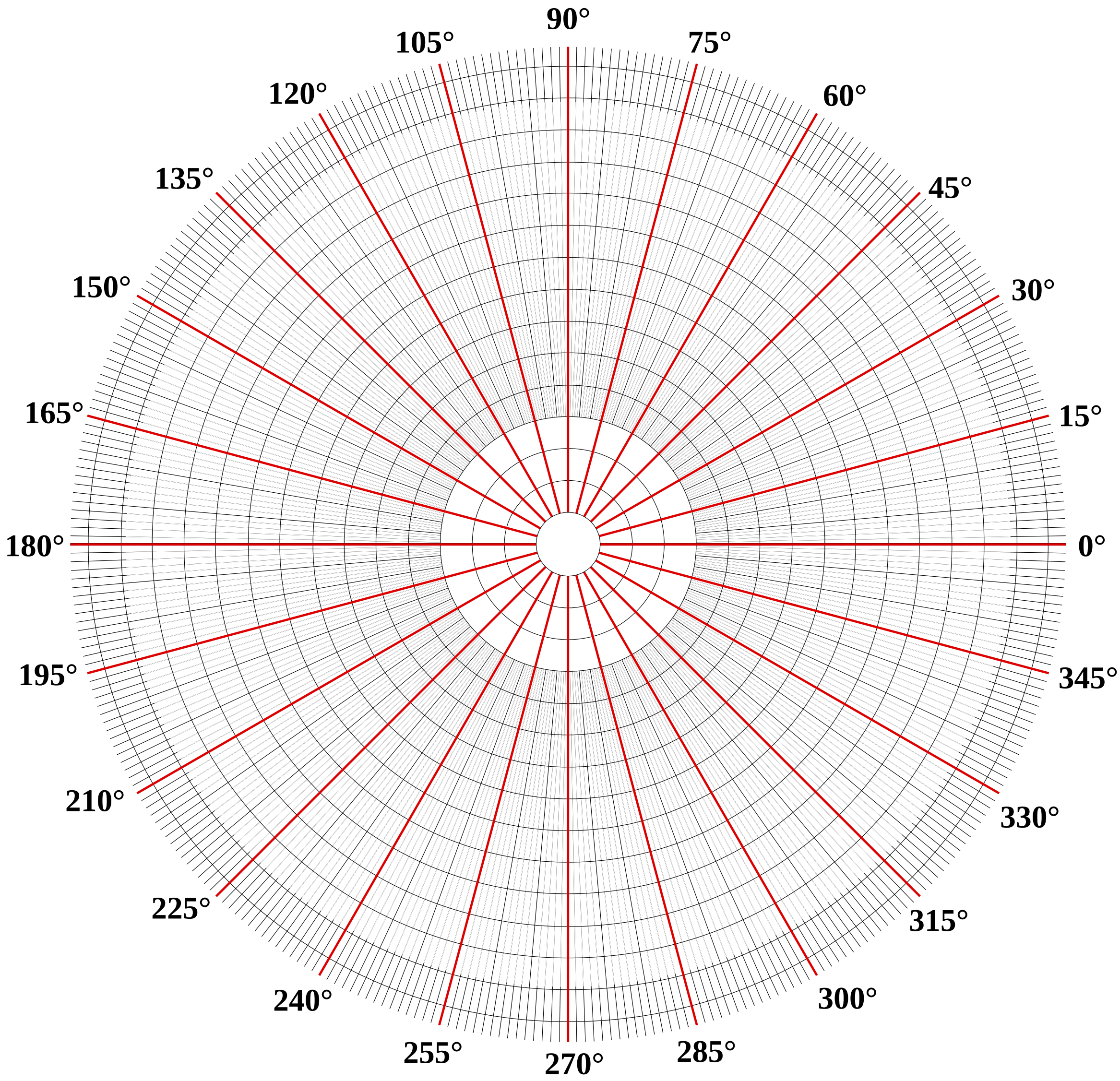
Work to 1 decimal place (and with your calculator in degrees mode !)

θ (in degrees)	0	45	90	100		135	165
$r = \dfrac{8}{2\cos\theta + \sin\theta}$							

θ (in degrees)	180	225	270	280		315	345
$r = \dfrac{8}{2\cos\theta + \sin\theta}$							

[4 marks]

(iii) Plot the polar coordinates from the table onto the graph below,



[4 marks]

(iv) This is an example of a polar curve of the form

$$r = \frac{c}{a\cos\theta + b\sin\theta} \quad \text{where } a,\ b \text{ and } c \text{ are constants}$$

"Reverse Engineer" this general equation to obtain the Cartesian equivalent.

[4 marks]

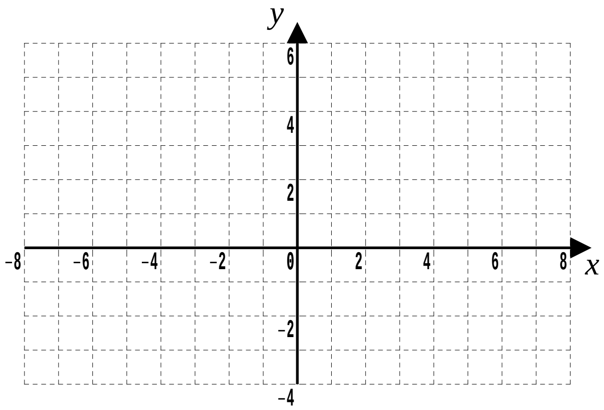
Question 2

(i) For the polar equation $r(\theta) = \dfrac{\theta}{90}$ complete the following table,

θ (in degrees)	90	180	270	360	450	540
$r = \dfrac{\theta}{90}$						

[2 marks]

(ii) Plot the graph of $r(\theta) = \dfrac{\theta}{90}$



[3 marks]

Question 3

Here is a polar equation which is to be graphed.

$$r(\theta) = 6\cos\theta + 8\sin\theta$$

(i) Explain why $r(0)$ and $r(180)$ yield the same point on the graph

[2 marks]

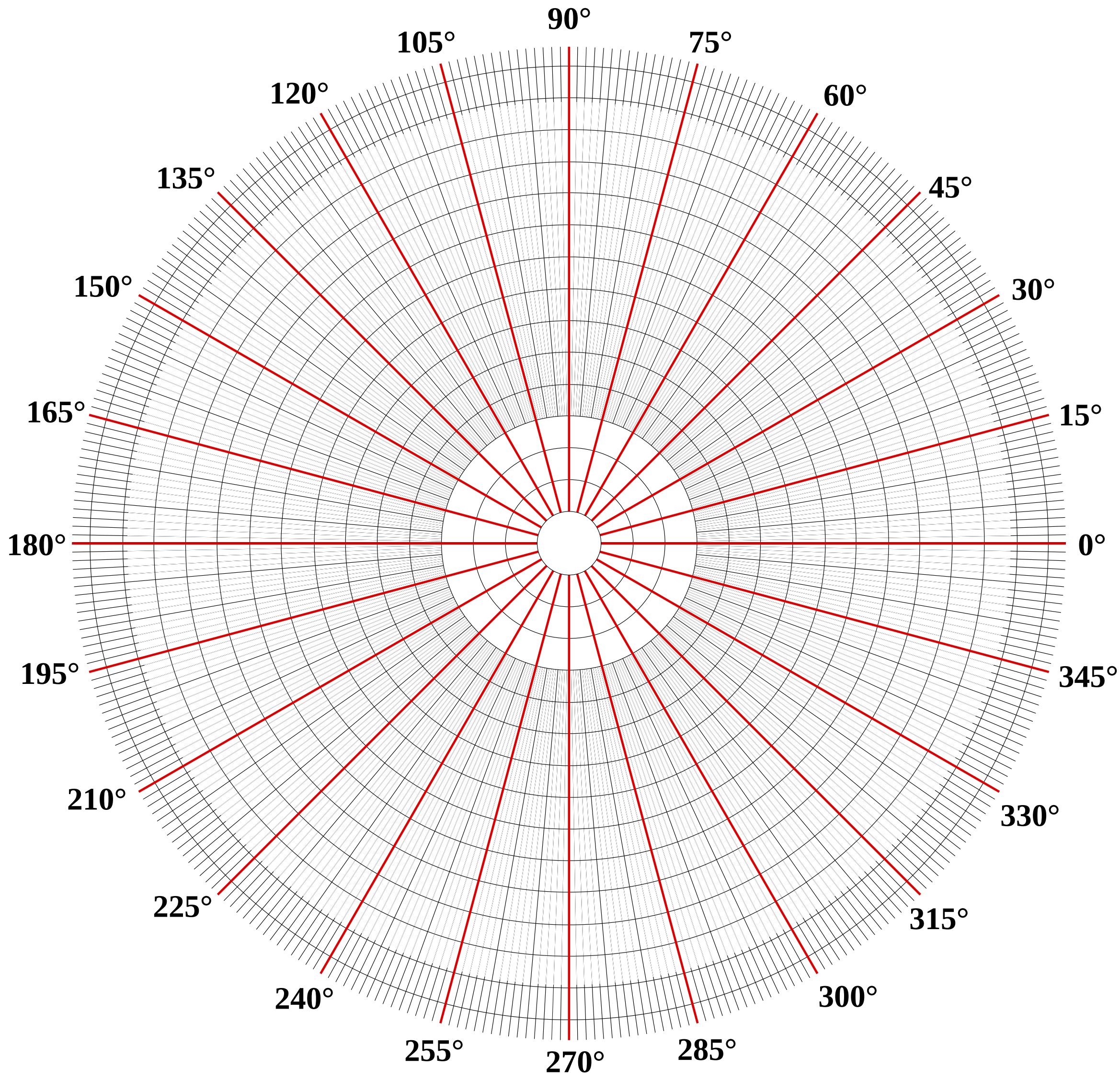
(ii) Complete the following table.
Notice the table is "intelligent" with a focus on key features.
(Plot extra points if you need to)
Work to 1 decimal place.

θ (in degrees)	0	10	20	30	45	75	90
$r = 6\cos\theta + 8\sin\theta$							

θ (in degrees)	100	110	120	140	180	210	270
$r = 6\cos\theta + 8\sin\theta$							

[5 marks]

(iii) Plot the polar coordinates from the table onto the graph below,



[5 marks]

(iv) This is an example of a polar curve of the form

$$r = 2a \cos\theta + 2b \sin\theta \qquad \text{where } a \text{ and } b \text{ are constants}$$

"Reverse Engineer" this general equation to obtain the Cartesian equivalent.

[5 marks]

Question 4

(i) Complete the following table for $r(\theta) = \dfrac{4}{1 - \sin\theta}$

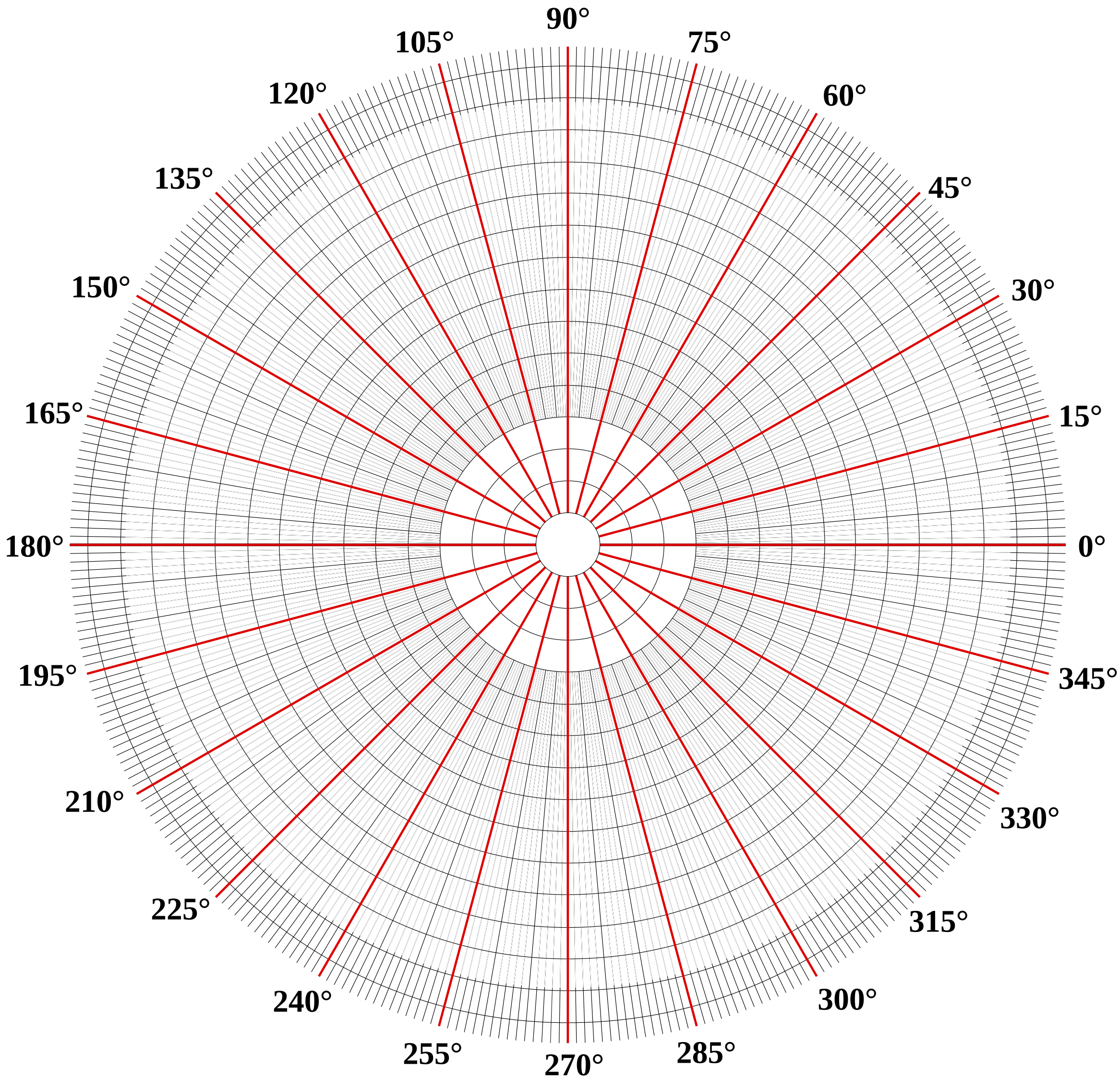
Work to 1 decimal place.

θ (in degrees)	0	15	30	45	[redacted]	135
$r = \dfrac{4}{1-\sin\theta}$						

θ (in degrees)	150	165	180	225	270	315
$r = \dfrac{4}{1-\sin\theta}$						

[4 marks]

(ii) Plot the polar coordinates from the table onto the graph below,



[4 marks]

(iii) This is an example of a polar curve of the form

$$r = \frac{p}{1 - \sin\theta} \qquad \text{where } p \text{ is a constant}$$

"Reverse Engineer" this general equation to obtain the Cartesian equivalent.

[6 marks]