6.1 Exam : Bring It ON

Polar equations, by their predominantly circular nature, are awash with trigonometric functions. In consequence, the resulting integrations will often involve squares of those functions especially when finding sector areas.

That's because of the r^2 in the formula $Area = \frac{1}{2} \int_{\alpha}^{\beta} r^2 \, d\theta$

Rather than memorising lots of results the prevailing wisdom is that it is better to focus on a few that are key, from which others are easily obtained when required.

6.2 Cosine

How to find $\int cos^2 \theta \, d\theta$

The two key results to use are;

$$cos^2 \theta + sin^2 \theta = 1$$
$$cos^2 \theta - sin^2 \theta = cos(2\theta)$$

Combine by addition;

$$2 \, cos^2 \theta = 1 + cos(2\theta)$$

$$\int cos^2 \theta \, d\theta = \frac{1}{2} \int 1 + cos(2\theta) \, d\theta$$

6.3 Sine

How to find $\int sin^2 \theta \, d\theta$

The two key results to use are;

$$cos^2 \theta + sin^2 \theta = 1$$
$$cos^2 \theta - sin^2 \theta = cos(2\theta)$$

Combine by subtraction;

$$2 \, sin^2 \theta = 1 - cos(2\theta)$$

$$\int sin^2 \theta \, d\theta = \frac{1}{2} \int 1 - cos(2\theta) \, d\theta$$

6.4 And Some...

$\int sec^2 \, d\theta = tan\, x + c$: Told this in examination formula booklet

$\int tan^2 \theta \, d\theta$: Use the identity $1 + tan^2 \theta = sec^2 \theta$

$\int csc^2 \theta \, d\theta = -cot\, \theta + c$: Told this in examination formula booklet

$\int cot^2 \theta \, d\theta$: Use the identity $cot^2 \theta + 1 = csc^2 \theta$

6.5 Exercise

Any solution based entirely on graphical
or numerical methods is not acceptable
Marks Available : 42

Question 1

Further A-Level Examination Question from June 2019, Core Paper 1, Q3 (Edexcel)

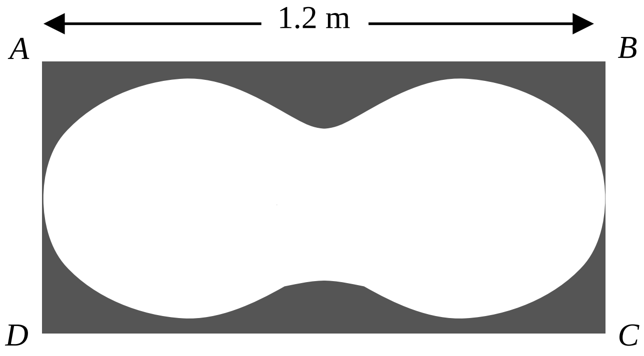


Diagram not to scale

The diagram shows the design for a table top in the shape of a rectangle *ABCD.*
The length of the table, *AB*, is 1.2 m.
The area inside the closed curve is made of glass.
The surrounding area, shown shaded, is made of wood.

The perimeter of the glass is modelled by the curve with polar equation

$$r = 0.4 + a\cos 2\theta \qquad 0 \leqslant \theta < 2\pi$$

where a is a constant.

(a) Show that $a = 0.2$

[2 marks]

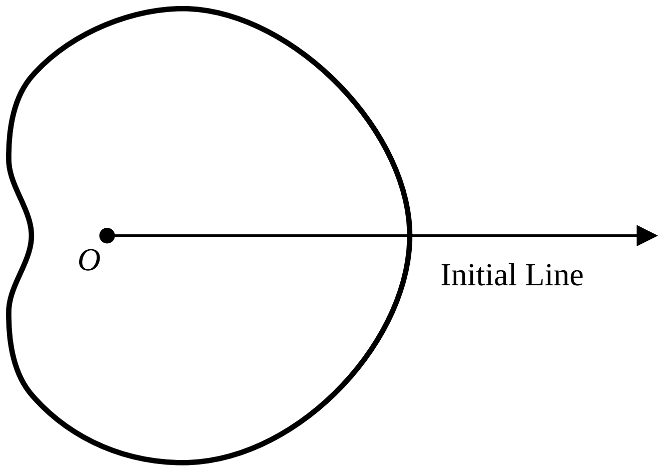
Hence, given that *AD* = 60 cm,

(b) find the area of the wooden part of the table top, giving your answer in m^2 to 3 significant figures.

[8 marks]

Question 2

Further A-Level Examination Question from June 2009, FP2, Q4 (Edexcel)



The sketch is of the curve with polar equation,

$$r = a + 3\cos\theta, \quad a > 0, \quad 0 \leqslant \theta < 2\pi$$

The area enclosed by the curve $\dfrac{107}{2}\pi$

Find the value of a

[8 marks]

Question 3

Further A-Level Examination Question from June 2015, FP2, Q1 (MEI)

(i) A curve has polar equation $r = 2a \cos\theta + 2b \sin\theta$,

where $a > 0$ and $b > 0$

Show, by considering the Cartesian equation, that the curve is a circle which passes through the origin.

Find the centre and radius of the circle in term of a and b

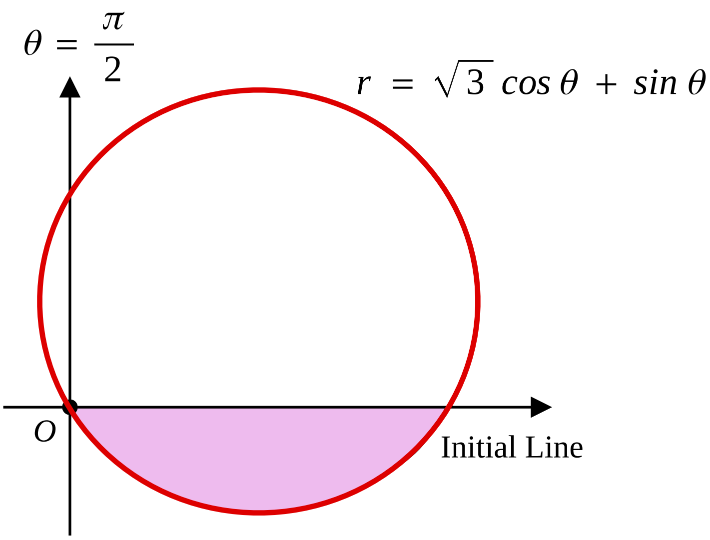
[5 marks]

(ii) For the case $a = b = 1$, use integration to show that the region bound by a minor arc of the circle and lines $\theta = \dfrac{\pi}{6}$ and $\theta = \dfrac{\pi}{3}$ has area $1 + \dfrac{\pi}{3}$

[5 marks]

Question 4

Further A-Level Examination Question, Paper O, FP2 (Madas Maths)



The diagram above shows the curve with polar equation

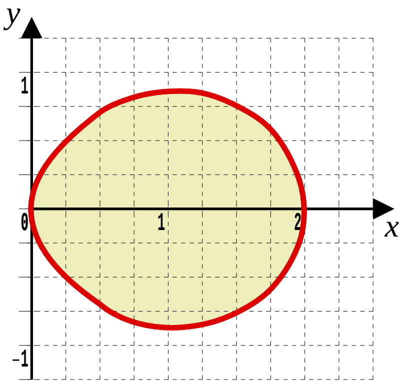
$$r = \sqrt{3}\cos\theta + \sin\theta, \quad -\frac{\pi}{3} \leqslant \theta < \frac{2\pi}{3}$$

By using a method involving integration in polar coordinates, show that the area of the shaded region is $\frac{1}{12}\left(4\pi - 3\sqrt{3}\right)$

[7 marks]

Question 5

Further A-Level Examination Question from Summer 2000, Mock P4, Q7 (Edexcel)



The curve C is given by; $\quad r = 2\cos^{\frac{3}{2}}\theta \qquad -\frac{\pi}{2} \leqslant \theta < \frac{\pi}{2}$

Find the area of the region enclosed by C

[7 marks]