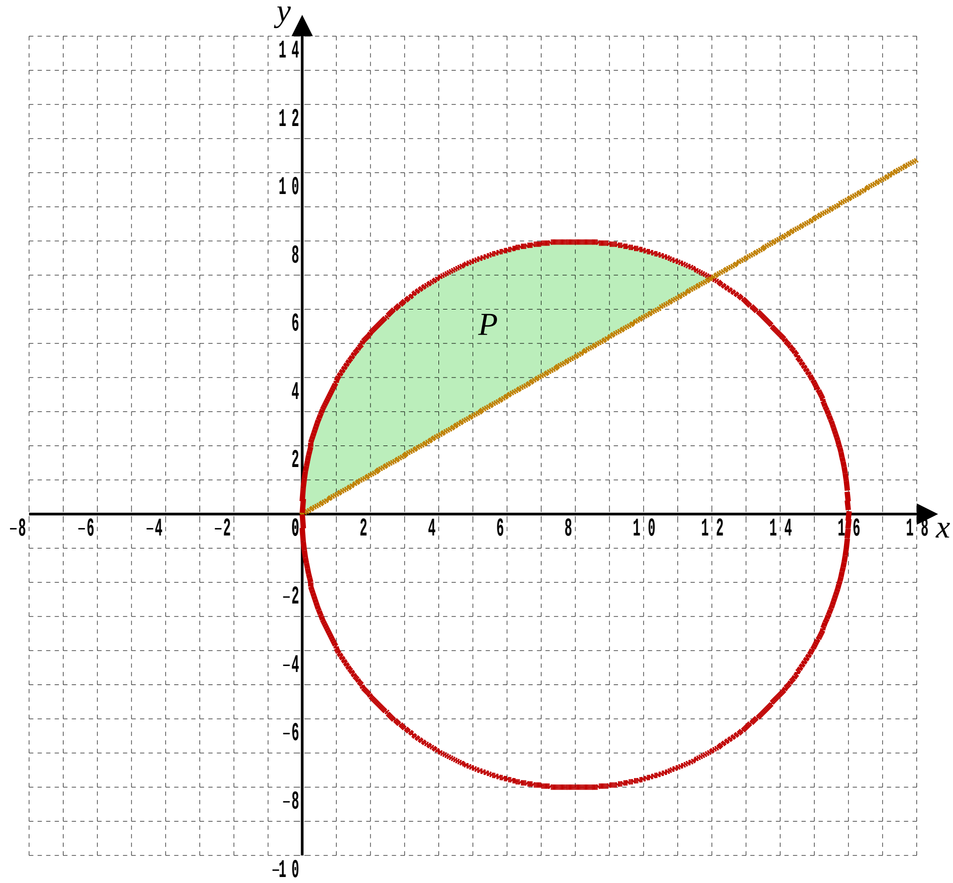
7.1 Area Problems, Taken Further

Here is a recap of the type of problem tackled in the previous lesson,



The segment shaded is in the circle $r = 16\cos\theta$ and bounded by $\theta = \dfrac{\pi}{6}$

It's area is given by,

$$P = \frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} 16^2 \cos^2\theta \, d\theta$$

$$= \frac{16^2}{2} \times \frac{1}{4}\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} (2 + 2\cos 2\theta)\, d\theta$$

$$= 32\,[\,2\theta + \sin 2\theta\,]_{\frac{\pi}{6}}^{\frac{\pi}{2}}$$

$$= 32\left[\pi + \sin\pi - \frac{\pi}{3} - \sin\left(\frac{\pi}{3}\right)\right]$$

$$= 32\left[\pi + 0 - \frac{\pi}{3} - \frac{\sqrt{3}}{2}\right]$$

$$= \frac{64\pi}{3} - 16\sqrt{3} \qquad \text{(About 39.3)}$$

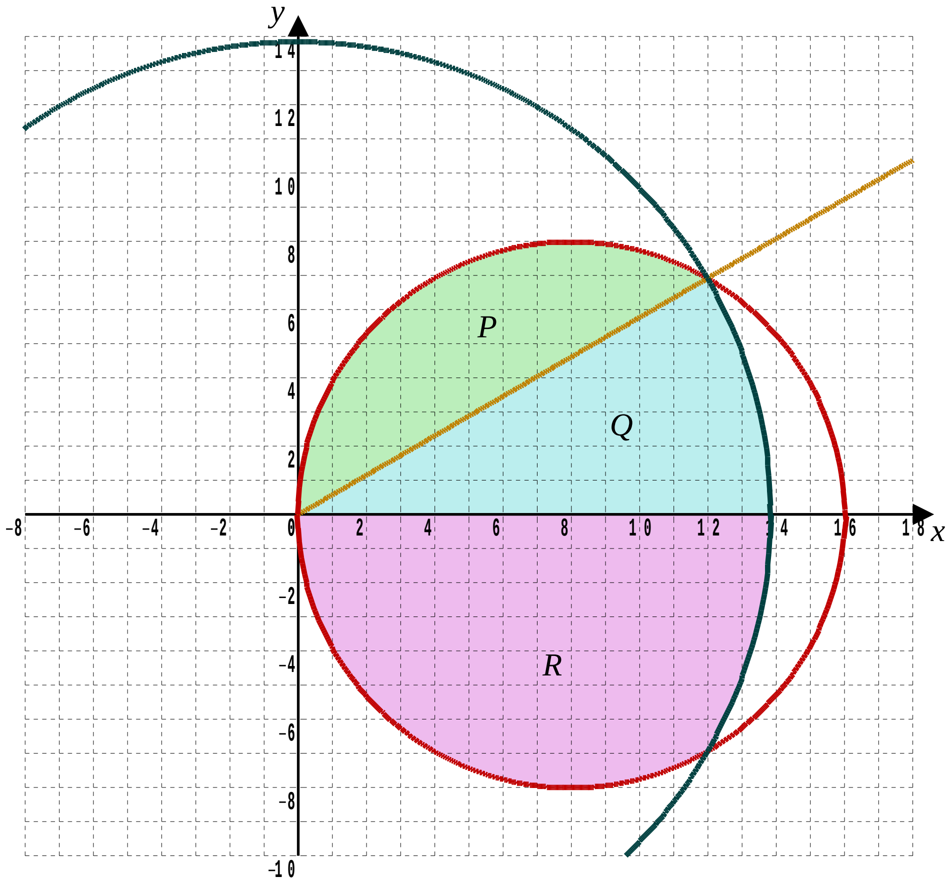
With this in mind, a related problem is considered next.

7.2 Area of Overlap

Find the area of the overlap between the two circles with polar equations,

$$r = 16\cos\theta \quad \text{and} \quad r = \sqrt{192}$$

This area is the total areas of the green, blue and red shaded regions below,



To find where the circles intersect,

$$16\cos\theta = \sqrt{192} \Rightarrow \cos\theta = \frac{\sqrt{3}}{2} \Rightarrow \theta = \pm\frac{\pi}{6}$$

Thus the gold line $\theta = \frac{\pi}{6}$ is added to the graph.

Half of the area sought is the green area, P, plus the blue area, Q.
But the green area, P, is the same as that worked out in the previous problem.

$$Q = \frac{1}{2}\int_0^{\frac{\pi}{6}} 192\, d\theta$$

$$= 16\pi$$

$$\therefore \text{ Area of overlap } = 2 \times (P + Q)$$

$$= 2\left(\frac{64\pi}{3} - 16\sqrt{3} + 16\pi\right)$$

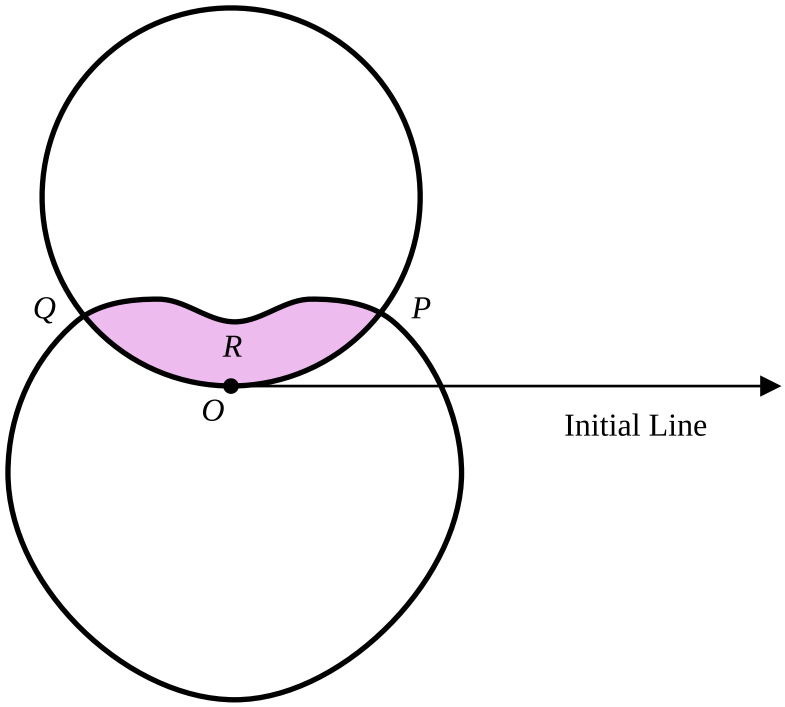
$$= \frac{224\pi}{3} - 32\sqrt{3} \qquad \text{(About 179)}$$

7.3 Exercise

Any solution based entirely on graphical
or numerical methods is not acceptable
Marks Available : 21

Question 1

Further A-Level Examination Question from June 2018, FP2, Q8 (Edexcel)



Th sketch is of the curves with polar equations

$$r = 2 \sin\theta \qquad 0 \leqslant \theta \leqslant \pi$$

$$r = 1.5 - \sin\theta \qquad 0 \leqslant \theta \leqslant 2\pi$$

The curves intersect at the points P and Q

(a) Find the polar coordinates of the point P and the polar coordinates of the point Q

[3 marks]

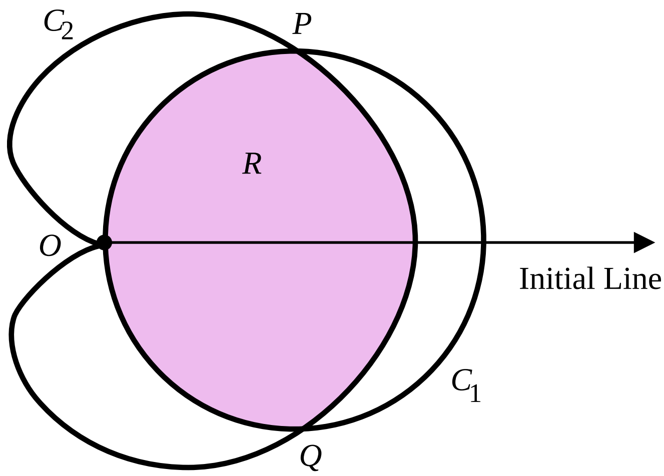
The region R, shown shaded, is enclosed by the two curves.

(b) Find the exact area of R giving your answer in the form $p\pi + q\sqrt{3}$, where p and q are rational numbers to be found.

[8 marks]

Question 2

Further A-Level Examination Question from June 2016, FP2, Q8 (Edexcel)



The sketch is of the curve C_1 with equation

$$r = 7\cos\theta \qquad -\frac{\pi}{2} < \theta \leqslant \frac{\pi}{2}$$

and the curve C_2 with equation

$$r = 3(1 + \cos\theta) \qquad -\pi < \theta \leqslant \pi$$

The curves C_1 and C_2 both pass through the pole and intersect at the point P and at the point Q

(a) Find the polar coordinates of P and the polar coordinates of Q

[3 marks]

The regions enclosed by the curve C_1 and the curve C_2 overlap, and the common region is shaded in the sketch.

(b) Find the area of R

[7 marks]