

Lesson 9

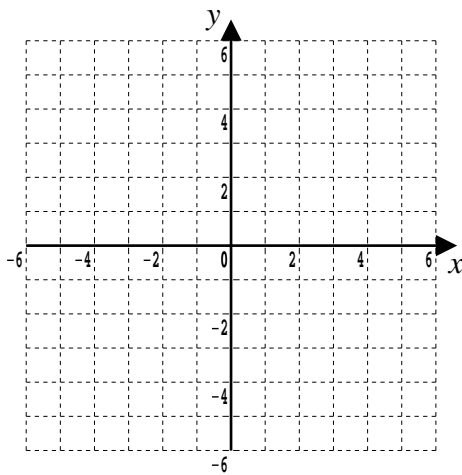
Further A-Level Pure Mathematics, Core 2  
**Polar Coordinates**

9.1 Revision

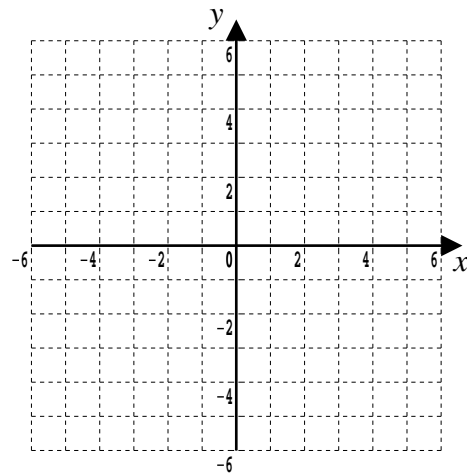
*Any solution based entirely on graphical  
or numerical methods is not acceptable*  
Marks Available : 40

**Question 1**

On each grid, sketch the graph of the polar equation given,

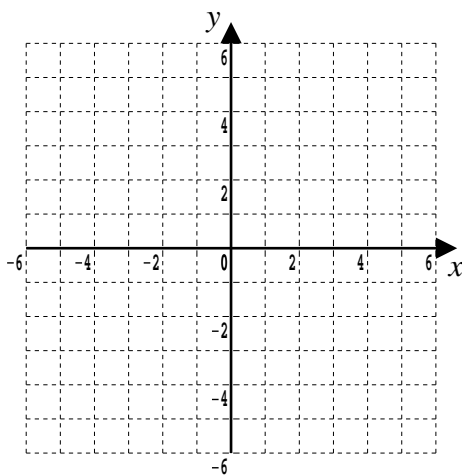


(i)  $r = 3 \csc \theta$

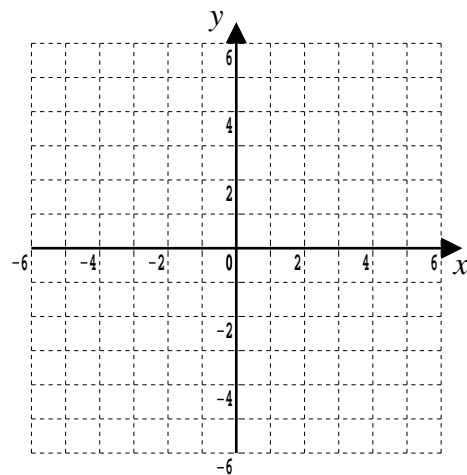


(ii)  $r = 3 \cos \theta$

[ 1, 1 marks ]



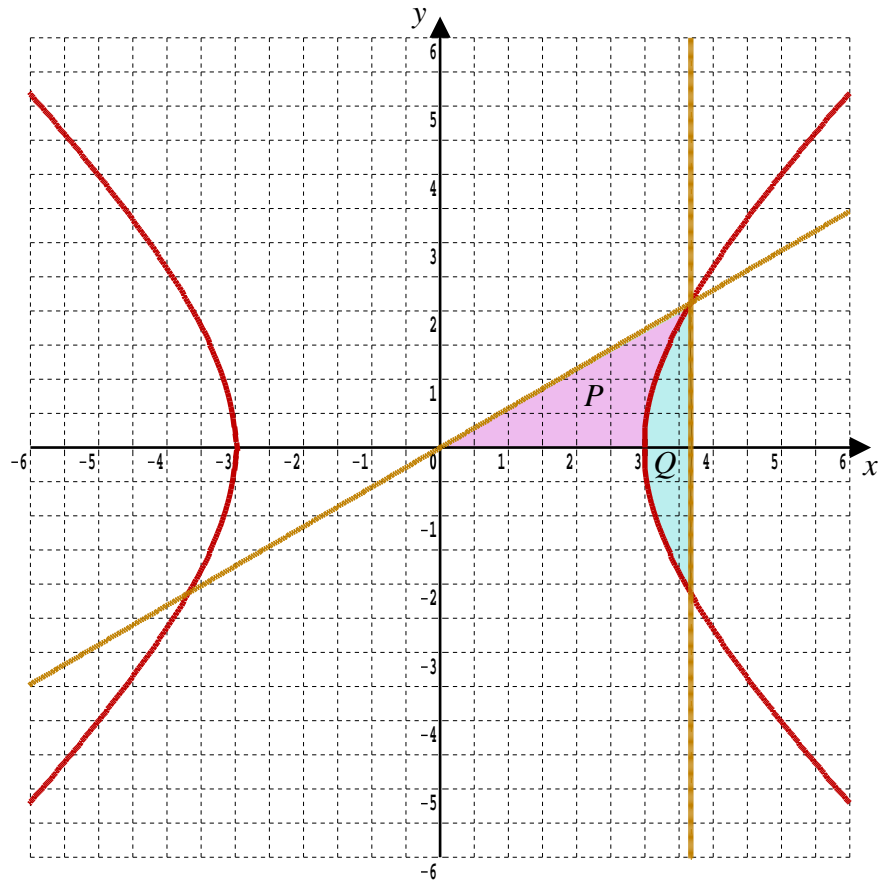
(iii)  $r = \frac{2}{\cos \theta + 2 \sin \theta}$



(iv)  $r = 4\sqrt{2} \sin (\theta - 45^\circ)$

[ 1, 1 marks ]

**Question 2**



The curve in the graph is of the hyperbola with polar equation  $r^2 = 9 \sec 2\theta$

Also shown is the vertical line with polar equation  $r = \frac{3\sqrt{6}}{2} \sec \theta$

- (i) Show that the curve and vertical line intersect when  $\theta = \pm \frac{\pi}{6}$

[ 4 marks ]

( ii ) The sloping straight line has polar equation  $\theta = \frac{\pi}{6}$

Show that the area shaded and marked  $P$  is  $\frac{9}{4} \ln(2 + \sqrt{3})$  by evaluating the following integral,

$$\text{Area } P = \frac{1}{2} \int_0^{\frac{\pi}{6}} 9 \sec 2\theta \, d\theta$$

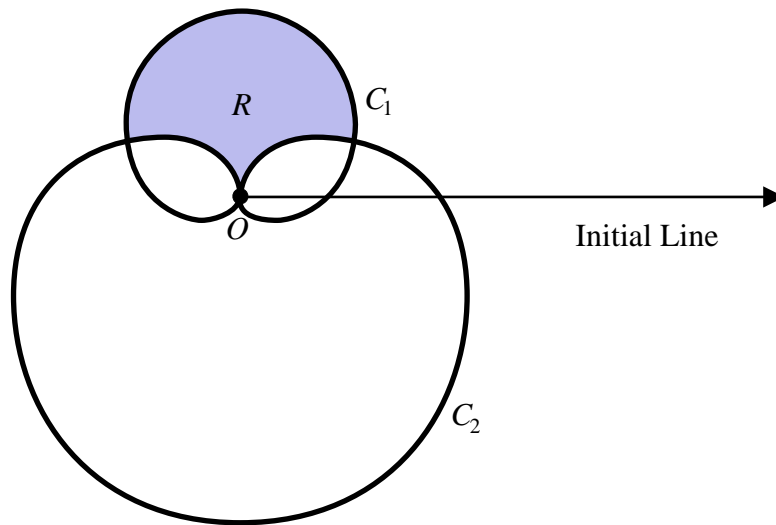
[ 5 marks ]

( iii ) Hence, or otherwise, find the exact area between the hyperbola and the vertical line which is shaded blue and marked  $Q$ .

[ 4 marks ]

**Question 3**

*Further A-Level Examination Question from October 2020, FP1, Q3 (Edexcel)*



The sketch is of two curves  $C_1$  and  $C_2$  with polar equations,

$$C_1 : r = (1 + \sin \theta) \quad 0 \leq \theta < 2\pi$$

$$C_2 : r = 3(1 - \sin \theta) \quad 0 \leq \theta < 2\pi$$

The region  $R$  lies inside  $C_1$  and outside  $C_2$  and is shown shaded in the sketch.

Show the area of  $R$  is  $p\sqrt{3} - q\pi$  where  $p$  and  $q$  are integers to be determined.

[ 9 marks ]

**Question 4**

*Further A-Level Examination Question from November 2021, FP2, Q6 (Edexcel)*

The curve  $C$  has polar equation  $r = a(p + 2 \cos \theta)$ ,  $0 \leq \theta < 2\pi$   
where  $a$  and  $p$  are positive constants and  $p > 2$ . There are exactly four points  
on  $C$  where the tangent is perpendicular to the initial line.

(a) Show that the range of possible values for  $p$  is  $2 < p < 4$

[ 5 marks ]

(b) Sketch the curve with equation  $r = a(3 + 2 \cos \theta)$ ,  $0 \leq \theta < 2\pi$   
where  $a > 0$

[ 1 mark ]

John digs a hole in his garden in order to make a pond.

The pond has a uniform horizontal cross section that is modelled by the curve  $r = 20(3 + 2 \cos \theta)$ ,  $0 \leq \theta < 2\pi$ , with  $r$  measured in centimetres.

The depth of the pond is 90 centimetres.

Water flows through a hosepipe into the pond at a rate of 50 litres per minute.

Given that the pond is initially empty,

( c ) determine how long it will take to completely fill the pond with water using the hosepipe, according to the model.

Give your answer to the nearest minute.

[ 7 marks ]

( d ) State a limitation of the model.

[ 1 mark ]

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Teachers may obtain detailed worked solutions to the exercises by email from [mhh@shrewsbury.org.uk](mailto:mhh@shrewsbury.org.uk)