

Lesson 11

Further A-Level Pure Mathematics, Core 2 Hyperbolic Functions

11.1 Revision

*Any solution based entirely on graphical
or numerical methods is not acceptable*

Marks Available : 40

Provided : Examination Mathematical Formulae Book

Question 1

Find $\int \sin^5 x \, dx$

[3 marks]

Question 2

Given that $y = \operatorname{artanh}(\cos x)$ find a simplified expression for $\frac{dy}{dx}$

[3 marks]

Question 3

Use the substitution $x = 5 \sinh u$ to show that, for some constant c ,

$$\int \sqrt{x^2 + 25} dx = \frac{1}{2} x \sqrt{x^2 + 25} + \frac{25}{2} \operatorname{arsinh}\left(\frac{x}{5}\right) + c$$

[5 marks]

Question 4

FM Examination Question from June 2021, Paper 1, Q4 (AQA)

Show that the solutions to the equation,

$$3 \tanh^2 x - 2 \operatorname{sech} x = 2$$

can be expressed in the form,

$$x = \pm \ln(a + \sqrt{b})$$

where a and b are integers to be found.

You may use without proof the result $\operatorname{arcosh} y = \ln\{y + \sqrt{y^2 - 1}\}$

[5 marks]

Question 5

Given that $y = \operatorname{arcosh} x$, show that for $x > 1$,

$$(x^2 - 1) \frac{d^3y}{dx^3} + 3x \frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$$

[7 marks]

Question 6

- (i) Use the substitution $x = \cosh^2 u$ to show that, for $x > 1$,

$$\int \sqrt{\frac{x}{x-1}} dx = \sqrt{x-1} \sqrt{x} + \operatorname{arcosh}(\sqrt{x}) + c$$

for some unknown constant, c

[6 marks]

- (ii) Hence, or otherwise, determine the exact value of $\int_2^4 \sqrt{\frac{x}{x-1}} dx$

You may use without proof the result $\operatorname{arcosh} y = \ln \{ y + \sqrt{y^2 - 1} \}$

[4 marks]

Question 7

Show that $\int_1^3 \frac{1}{\sqrt{3x^2 - 6x + 7}} dx = \frac{1}{\sqrt{3}} \ln(2 + \sqrt{3})$

[7 marks]

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Teachers may obtain detailed worked solutions to the exercises by email from mhh@shrewsbury.org.uk