Lesson 11

Further A-Level Pure Mathematics, Core 2

Hyperbolic Functions

11.1 Revision

Any solution based entirely on graphical
or numerical methods is not acceptable

Marks Available : 40

Provided : Examination Mathematical Formulae Book

Question 1

Find $\int sin^5 x \, dx$

[3 marks]

Question 2

Given that $y = artanh(cos\, x)$ find a simplified expression for $\frac{dy}{dx}$

[3 marks]

Question 3

Use the substitution $x = 5 \sinh u$ to show that, for some constant c,

$$\int \sqrt{x^2 + 25}\, dx = \frac{1}{2} x \sqrt{x^2 + 25} + \frac{25}{2} \operatorname{arsinh}\left(\frac{x}{5}\right) + c$$

[5 marks]

Question 4

FM Examination Question from June 2021, Paper 1, Q4 (AQA)

Show that the solutions to the equation,

$$3\tanh^2 x - 2\,\mathrm{sech}\, x = 2$$

can be expressed in the form,

$$x = \pm \ln\left(a + \sqrt{b}\right)$$

where a and b are integers to be found.

You may use without proof the result $\operatorname{arcosh} y = \ln\left\{y + \sqrt{y^2 - 1}\right\}$

[5 marks]

Question 5

Given that $y = \operatorname{arcosh} x$, show that for $x > 1$,

$$\left(x^2 - 1\right)\frac{d^3y}{dx^3} + 3x\frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$$

[7 marks]

Question 6

(i) Use the substitution $x = \cosh^2 u$ to show that, for $x > 1$,

$$\int \sqrt{\frac{x}{x-1}}\, dx = \sqrt{x-1}\,\sqrt{x} + \operatorname{arcosh}\left(\sqrt{x}\right) + c$$

for some unknown constant, c

[6 marks]

(ii) Hence, or otherwise, determine the exact value of $\int_2^4 \sqrt{\frac{x}{x-1}}\, dx$

You may use without proof the result $\operatorname{arcosh} y = \ln\left\{ y + \sqrt{y^2 - 1} \right\}$

[4 marks]

Question 7

Show that $\int_1^3 \frac{1}{\sqrt{3x^2 - 6x + 7}}\, dx = \frac{1}{\sqrt{3}} ln\left(2 + \sqrt{3}\right)$

[7 marks]