2.1 Identities

The hyperbolic functions have identities that resonate with those already known for the trigonometric functions. For example there are hyperbolic addition formulae, hyperbolic double angle formulae and relationships between the squares of the hyperbolic functions that have echos with their trigonometric counterparts. Initially, hyperbolic identities are proven from the definitions in terms of exponentials but as a repertoire of relationships is built up many short cuts become available.

2.2 Example

Prove that $\cosh^2 x - \sinh^2 x = 1$

Teaching Video: http://www.NumberWonder.co.uk/v9102/2.mp4

[4 marks]

2.3 Exercise

Any solution based entirely on graphical or numerical methods is not acceptable

Marks Available : 30

Question 1

Prove that $\cosh^2 x + \sinh^2 x = \cosh 2x$

[4 marks]

Question 2

Prove that $2\,sinh\,x\;\;cosh\,x \;=\; sinh\,2x$

[4 marks]

Question 3

Given that $sinh\,x \;=\; \dfrac{3}{4}$ and by using the identities proven in the example and question 1 and question 2, find the exact value of,

(i) $cosh\,x$

[2 marks]

(ii) $tanh\,x$

[1 mark]

(iii) $sinh\,2x$

[1 mark]

(iv) $cosh\,2x$

[2 marks]

(v) $sinh\,4x$

[1 mark]

Question 4

Prove the following identities using the definitions of *cosh x* and *sinh x*

Top Tip : Start with the RHS and work towards the LHS

(i) $cosh\,(A + B) = cosh\,A\,cosh\,B + sinh\,A\,sinh\,B$

[5 marks]

(ii) $cosh\,3A = 4\,cosh^3 A - 3\,cosh\,A$

[5 marks]

(iii) $sinh\,A - sinh\,B = 2sinh\left(\frac{A - B}{2}\right)cosh\left(\frac{A + B}{2}\right)$

[5 marks]