

Lesson 2

Further A-Level Pure Mathematics, Core 2 Hyperbolic Functions

2.1 Identities

The hyperbolic functions have identities that resonate with those already known for the trigonometric functions. For example there are hyperbolic addition formulae, hyperbolic double angle formulae and relationships between the squares of the hyperbolic functions that have echoes with their trigonometric counterparts. Initially, hyperbolic identities are proven from the definitions in terms of exponentials but as a repertoire of relationships is built up many short cuts become available.

2.2 Example

Prove that $\cosh^2 x - \sinh^2 x = 1$

Teaching Video: <http://www.NumberWonder.co.uk/v9102/2.mp4>



[4 marks]

2.3 Exercise

*Any solution based entirely on graphical
or numerical methods is not acceptable*

Marks Available : 30

Question 1

Prove that $\cosh^2 x + \sinh^2 x = \cosh 2x$

[4 marks]

Question 2

Prove that $2 \sinh x \cosh x = \sinh 2x$

[4 marks]

Question 3

Given that $\sinh x = \frac{3}{4}$ and by using the identities proven in the example and question 1 and question 2, find the exact value of,

(i) $\cosh x$

[2 marks]

(ii) $\tanh x$

[1 mark]

(iii) $\sinh 2x$

[1 mark]

(iv) $\cosh 2x$

[2 marks]

(v) $\sinh 4x$

[1 mark]

Question 4

Prove the following identities using the definitions of $\cosh x$ and $\sinh x$

Top Tip : Start with the RHS and work towards the LHS

(i) $\cosh(A + B) = \cosh A \cosh B + \sinh A \sinh B$

[5 marks]

(ii) $\cosh 3A = 4 \cosh^3 A - 3 \cosh A$

[5 marks]

(iii) $\sinh A - \sinh B = 2\sinh\left(\frac{A - B}{2}\right)\cosh\left(\frac{A + B}{2}\right)$

[5 marks]

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Teachers may obtain detailed worked solutions to the exercises by email from mhh@shrewsbury.org.uk