

Lesson 6

Further A-Level Pure Mathematics, Core 2 Hyperbolic Functions

6.1 Differentiation

The hyperbolic functions and their inverse functions are differentiable. As no new ideas are involved this provides a welcome opportunity to refresh the various differentiation techniques of A-Level and Further A-Level mathematics.

6.2 Table of standard derivatives (Hyperbolic Functions)

$f(x)$	$f'(x)$	In Formula Book ?
$\sinh x$	$\cosh x$	Yes
$\cosh x$	$\sinh x$	Yes
$\tanh x$	$\operatorname{sech}^2 x$	Yes
$\operatorname{arsinh} x$	$\frac{1}{\sqrt{x^2+1}}$	Yes
$\operatorname{arcosh} x$	$\frac{1}{\sqrt{x^2-1}} \quad x > 1$	Yes
$\operatorname{artanh} x$	$\frac{1}{1-x^2} \quad x < 1$	Yes

6.3 Exercise

*Any solution based entirely on graphical
or numerical methods is not acceptable*

Marks Available : 30

Question 1

Differentiate with respect to x ,

(i) $y = \sinh(5x)$

[1 mark]

(ii) $y = \tanh\left(\frac{x}{3}\right)$

[1 mark]

(iii) $y = \operatorname{arcosh}(2x)$

[1 mark]

(iv) $y = \operatorname{arsinh}\left(\frac{x}{3}\right)$

[2 mark]

Question 2

If $y = a \cosh(nx) + b \sinh(nx)$ where a and b are constants, prove that,

$$\frac{d^2y}{dx^2} = n^2 y$$

[4 marks]

Question 3

Given that $y = (\operatorname{arcosh} x)^2$ prove that

$$(x^2 - 1) \left(\frac{dy}{dx} \right)^2 = 4y$$

[5 marks]

Question 4

Differentiate with respect to x ,

(i) $y = \sinh(2x) \cosh(3x)$

[2 marks]

(ii) $y = \frac{\cosh x}{4x}$

[2 marks]

(iii) $y = x^2 \operatorname{arcosh} x$

[2 marks]

Question 5

Further A-Level Examination Question from June 2012, FP3, Q5(a) (Edexcel)

Differentiate $y = x \operatorname{arsinh}(2x)$ with respect to x

[3 marks]

Question 6

Differentiate with respect to x ,

$$y = \operatorname{sech}(2x)$$

[3 marks]

Question 7

Further A-Level Examination Question from June 2014, FP3, Q5 (Edexcel)

Given that $y = \operatorname{artanh}\left(\frac{x}{\sqrt{1+x^2}}\right)$ show that $\frac{dy}{dx} = \frac{1}{\sqrt{1+x^2}}$

[4 marks]

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Teachers may obtain detailed worked solutions to the exercises by email from mhh@shrewsbury.org.uk