6.1 Differentiation

The hyperbolic functions and their inverse functions are differentiable. As no new ideas are involved this provides a welcome opportunity to refresh the various differentiation techniques of A-Level and Further A-Level mathematics.

6.2 Table of standard derivatives (Hyperbolic Functions)

$f(x)$	$f'(x)$	In Formula Book ?
$\sinh x$	$\cosh x$	Yes
$\cosh x$	$\sinh x$	Yes
$\tanh x$	$\text{sech}^2 x$	Yes
$\text{arsinh}\ x$	$\dfrac{1}{\sqrt{x^2+1}}$	Yes
$\text{arcosh}\ x$	$\dfrac{1}{\sqrt{x^2-1}} \quad x > 1$	Yes
$\text{artanh}\ x$	$\dfrac{1}{1-x^2} \quad \lvert x \rvert < 1$	Yes

6.3 Exercise

Any solution based entirely on graphical or numerical methods is not acceptable

Marks Available : 30

Question 1

Differentiate with respect to *x*,

(i) $y = \sinh(5x)$

[1 mark]

(ii) $y = \tanh\left(\dfrac{x}{3}\right)$

[1 mark]

(iii) $y = \text{arcosh}(2x)$

[1 mark]

(iv) $y = \text{arsinh}\left(\dfrac{x}{3}\right)$

[2 mark]

Question 2

If $y = a\cosh(nx) + b\sinh(nx)$ where *a* and *b* are constants, prove that,

$$\frac{d^2y}{dx^2} = n^2 y$$

[4 marks]

Question 3

Given that $y = (\text{arcosh}\, x)^2$ prove that

$$\left(x^2 - 1\right)\left(\frac{dy}{dx}\right)^2 = 4y$$

[5 marks]

Question 4
Differentiate with respect to *x*,

(i) $y = sinh(2x)\ cosh(3x)$

[2 marks]

(ii) $y = \dfrac{cosh\, x}{4x}$

[2 marks]

(iii) $y = x^2\ arcosh\, x$

[2 marks]

Question 5
Further A-Level Examination Question from June 2012, FP3, Q5(a) (Edexcel)
Differentiate $y = x\ arsinh(2x)$ with respect to x

[3 marks]

Question 6

Differentiate with respect to *x*,

$$y = sech\left(2x\right)$$

[3 marks]

Question 7

Further A-Level Examination Question from June 2014, FP3, Q5 (Edexcel)

Given that $y = artanh\left(\dfrac{x}{\sqrt{1 + x^2}}\right)$ show that $\dfrac{dy}{dx} = \dfrac{1}{\sqrt{1 + x^2}}$

[4 marks]