8.1 Integration (Chain Rule Backwards)

In principle, given we've already[†] looked at differentiation questions, tackling an exercise on integration is just a case of using the following, previously supplied, table backwards, and remembering that if it's an indefinite integral (one without limits) to put in the constant of integration, c.

$f(x)$	$f'(x)$	In Formula Book ?
$\sinh x$	$\cosh x$	Yes
$\cosh x$	$\sinh x$	Yes
$\tanh x$	$\operatorname{sech}^2 x$	Yes
$\operatorname{arsinh} x$	$\frac{1}{\sqrt{x^2+1}}$	Yes
$\operatorname{arcosh} x$	$\frac{1}{\sqrt{x^2-1}} \quad x > 1$	Yes
$\operatorname{artanh} x$	$\frac{1}{1-x^2} \quad \lvert x \rvert < 1$	Yes

As with any non-trivial integration, always scan straight away for a possible "Chain Rule Backwards" before reaching for the much bigger weapons in the armoury; Integration by Substitution and Integration by Parts.

8.2 Example

Find $\displaystyle\int \frac{2 + 5x}{\sqrt{x^2 + 1}}\, dx$

Teaching Video : http://www.NumberWonder.co.uk/v9102/8.mp4

[6 marks]

† Hyperbolic Functions, Lesson 6

8.3 Exercise

Any solution based entirely on graphical or numerical methods is not acceptable

Marks Available : 50

Question 1

Integrate the following with respect to *x*,

(i) $\sinh(5x)$ **(ii)** $\dfrac{1}{\cosh^2(4x)}$

[2, 3 marks]

Question 2

Marvin is thinking of using integration by parts twice to find the following integral,

$$\int x^2 \cosh\left(x^3 + 4\right)\, dx$$

but Giles points out that it can far more easily be done as a “chain rule backwards”. Following Giles' advice, carry out the integration.

[2 marks]

Question 3

By setting up a “chain rule backwards” find,

(i) $\int \sinh(5x)\, \cosh^6(5x)\, dx$

(ii) $\int \dfrac{\sinh(5x)}{\cosh^6(5x)}\, dx$

[4 marks]

Question 4

Frederic wishes to integrate the *tanh x* function and has begun by setting up a "chain rule backwards".

(i) Complete Fredric's solution,

$$\int \tanh x \, dx = \int \frac{\sinh x}{\cosh x} \, dx$$

$$= \int \sinh x \, (\cosh x)^{-1} \, dx$$

[2 marks]

(ii) Justify why no modulus signs are needed in the final result.

[1 mark]

Question 5

(i) Use the approach outlined in question 4 to find $\int \coth x \, dx$

[2 marks]

(ii) Are modulus signs are needed in the final result ?

[1 mark]

Question 6

Find the exact value of $\int_0^{\sqrt{2}} \frac{6}{\sqrt{1 + 4x^2}} \, dx$

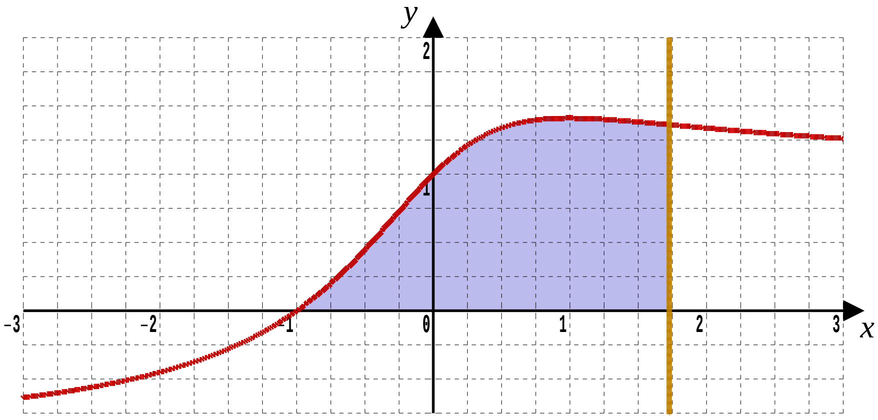
[5 marks]

Question 7

The graph is of the function $f(x) = \dfrac{1 + x}{\sqrt{x^2 + 1}}$ (in red) along with the vertical line (in gold) with equation $x = \sqrt{3}$. Show that the exact value of the area bounded by this vertical line, $f(x)$, and the x-axis is,

$$ln\left(\left(\sqrt{3} + 2\right)\left(\sqrt{2} + 1\right)\right) + 2 - \sqrt{2}$$

The techniques of example 8.2 will be useful !



[8 marks]

Question 8

By replacing the $\cosh x$ function with exponentials, find $\int_0^{\ln 3} e^x \cosh(2x)\; dx$

[4 marks]

Question 9

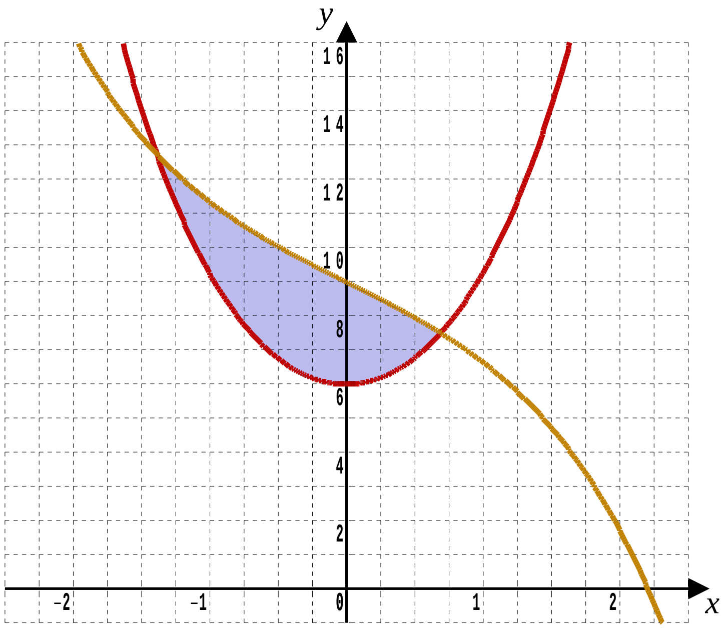
FM A-Level Examination Question from June 2014, Paper FP3, Q3(b) (Edexcel)

Using calculus, find the exact value of $\int_0^1 e^{2x} \sinh x \; dx$

[4 marks]

Question 10

FM A-Level Examination Question from June 2013, Paper FP3, Q7 (Edexcel)



The curves shown are $y = 6\cosh x$ (red) and $y = 9 - 2\sinh x$ (gold).

(a) Using the definitions of $\sinh x$ and $\cosh x$ in terms of e^x, find exact values for the x-coordinate of the two points where the curves intersect.

[6 marks]

The finite region between the two curves is shown shaded.

(b) Using calculus, find the area of the shaded region, giving your answer in the form $a \ln b + c$, where a, b and c are integers.

[6 marks]