



Fortify Your Maths

Any solution based entirely on graphical or numerical methods is not acceptable

Marks Available : 30

Question 1

The matrix $\mathbf{A} = \begin{pmatrix} 3 & k & 0 \\ -2 & 1 & 2 \\ 5 & 0 & k + 3 \end{pmatrix}$, where k is a constant

(i) Find $\det \mathbf{A}$ in terms of k

[2 marks]

Given that \mathbf{A} is singular,

(ii) find the possible values of k

[2 marks]

Question 2

$$f(z) = z^3 + 3z^2 + kz + 48, \quad k \in \mathbb{R}$$

Given that $f(4i) = 0$

(a) find the value of k

[2 marks]

(b) find and list all the roots of the equation

[3 marks]

Question 3

Show that $\frac{\cos 2x + i \sin 2x}{\cos 9x - i \sin 9x}$ can be expressed in the form $\cos nx + i \sin nx$,
where n is an integer to be found

[4 marks]

Question 4

Given that $\sum_{r=1}^n r^2(r-1) = \frac{1}{12}n(n+1)(pn^2 + qn + r)$

(a) find the values of p , q and r

[4 marks]

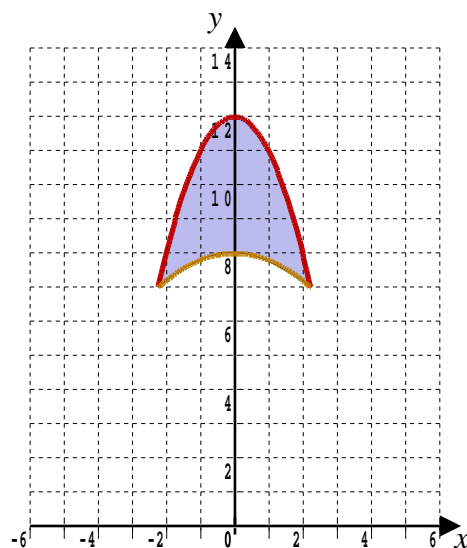
(b) Hence evaluate $\sum_{r=50}^{100} r^2(r-1)$

[2 marks]

Question 5

The diagram shows parts of the curves with equations,

$y = 12 - x^2$ (in red) and $y = 8 - 0.2x^2$ (in gold)



A jeweller models a gold ring as the volume of revolution formed when the area bounded by these two curves is rotated through 360° about the x -axis

- (i) Given that the dimensions on the diagram are in millimetres, state the maximum outer diameter of the ring

[1 mark]

The density of gold is 19.3 g cm^{-3}

- (ii) Find the mass of the ring according to this model, giving your answer in grams to 1 decimal place.

[9 marks]

- (iii) Give one reason why the actual mass of the ring is likely to be different from your answer to part (ii).

[1 mark]

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Teachers may obtain detailed worked solutions to the exercises by email from mhh@shrewsbury.org.uk