Fortify Your Maths

Any solution based entirely on graphical
or numerical methods is not acceptable
Marks Available : 30

Question 1

The matrix $\mathbf{A} = \begin{pmatrix} 3 & k & 0 \\ -2 & 1 & 2 \\ 5 & 0 & k+3 \end{pmatrix}$, where k is a constant

(**i**)   Find   *det* **A**   in terms of k

[**2 marks**]

Given that **A** is singular,
(**ii**)   find the possible values of k

[**2 marks**]

Question 2

$$f(z) = z^3 + 3z^2 + kz + 48, \ k \in \mathbb{R}$$

Given that $f(4i) = 0$

(**a**) find the value of k

[2 marks]

(**b**) find and list all the roots of the equation

[3 marks]

Question 3

Show that $\dfrac{\cos 2x + i \sin 2x}{\cos 9x - i \sin 9x}$ can be expressed in the form $\cos nx + i \sin nx$,

where n is an integer to be found

[4 marks]

Question 4

Given that $\displaystyle\sum_{r=1}^{n} r^2(r-1) = \frac{1}{12}n(n+1)(pn^2 + qn + r)$

(**a**)   find the values of p, q and r

[4 marks]

(**b**)   Hence evaluate $\displaystyle\sum_{r=50}^{100} r^2(r-1)$

[2 marks]

Question 5

The diagram shows parts of the curves with equations,

$$y = 12 - x^2 \quad \text{(in red)} \quad \text{and} \quad y = 8 - 0.2\,x^2 \quad \text{(in gold)}$$
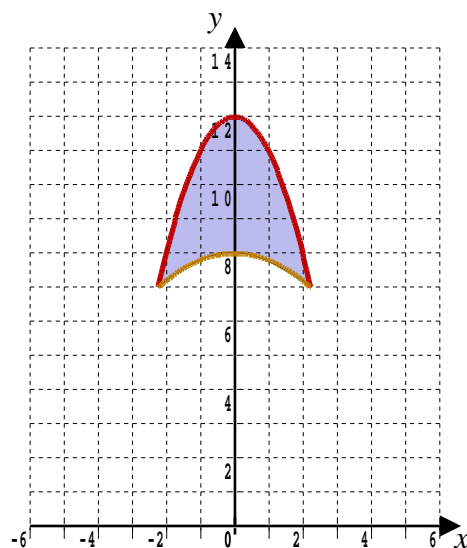
A jeweller models a gold ring as the volume of revolution formed when the area bounded by these two curves is rotated through 360° about the *x*-axis

(**i**) Given that the dimensions on the diagram are in millimetres, state the maximum outer diameter of the ring

[1 mark]

The density of gold is 19.3 g cm^{-3}

(**ii**) Find the mass of the ring according to this model, giving your answer in grams to 1 decimal place.

[9 marks]

(**iii**) Give one reason why the actual mass of the ring is likely to be different from your answer to part (ii).

[1 mark]