Year FM Further Pure Mathematics Examination Revision : Health Check N° 5

A year ago the Doctor told me I was going deaf. I haven't heard from him since.

Any solution based entirely on graphical or numerical methods is not acceptable

Marks Available : 30

Question 1

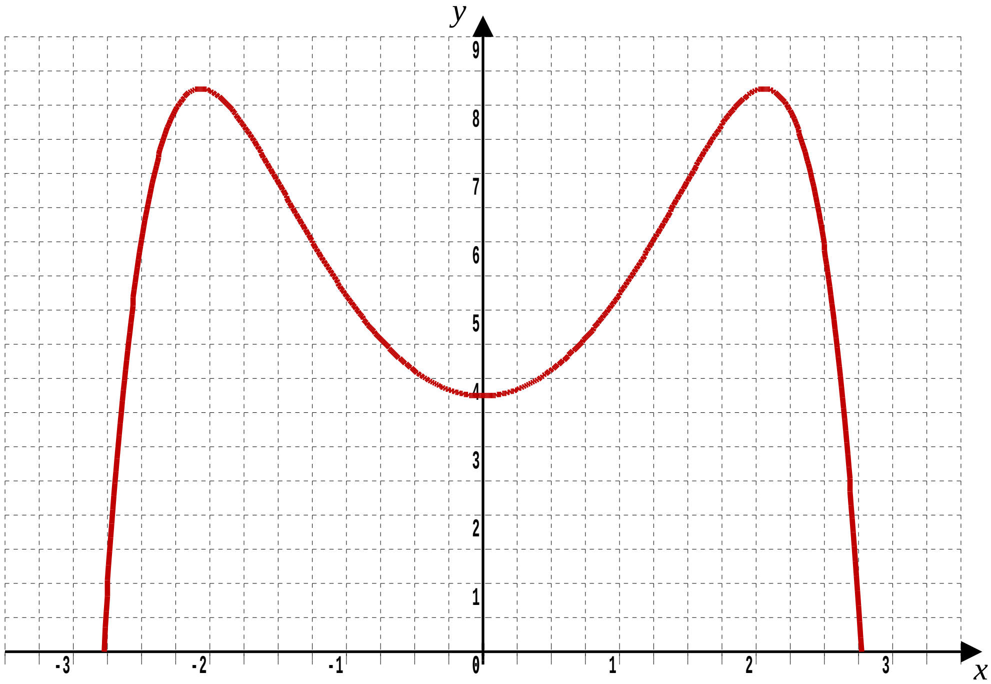
Find $\int \frac{1}{4 + 3x^2}\, dx$ giving your answer in the form $A \arctan(Bx) + c$ where c is an arbitrary constant and A and B are constants to be found.

[4 marks]

Question 2

The graph is of the function $f(x) = 4\cosh x - \frac{1}{4}\cosh(2x), \quad x \in \mathbb{R}$

Determine the exact x coordinates of the stationary points.
Your answer should be in terms of natural logarithms.



[5 marks]

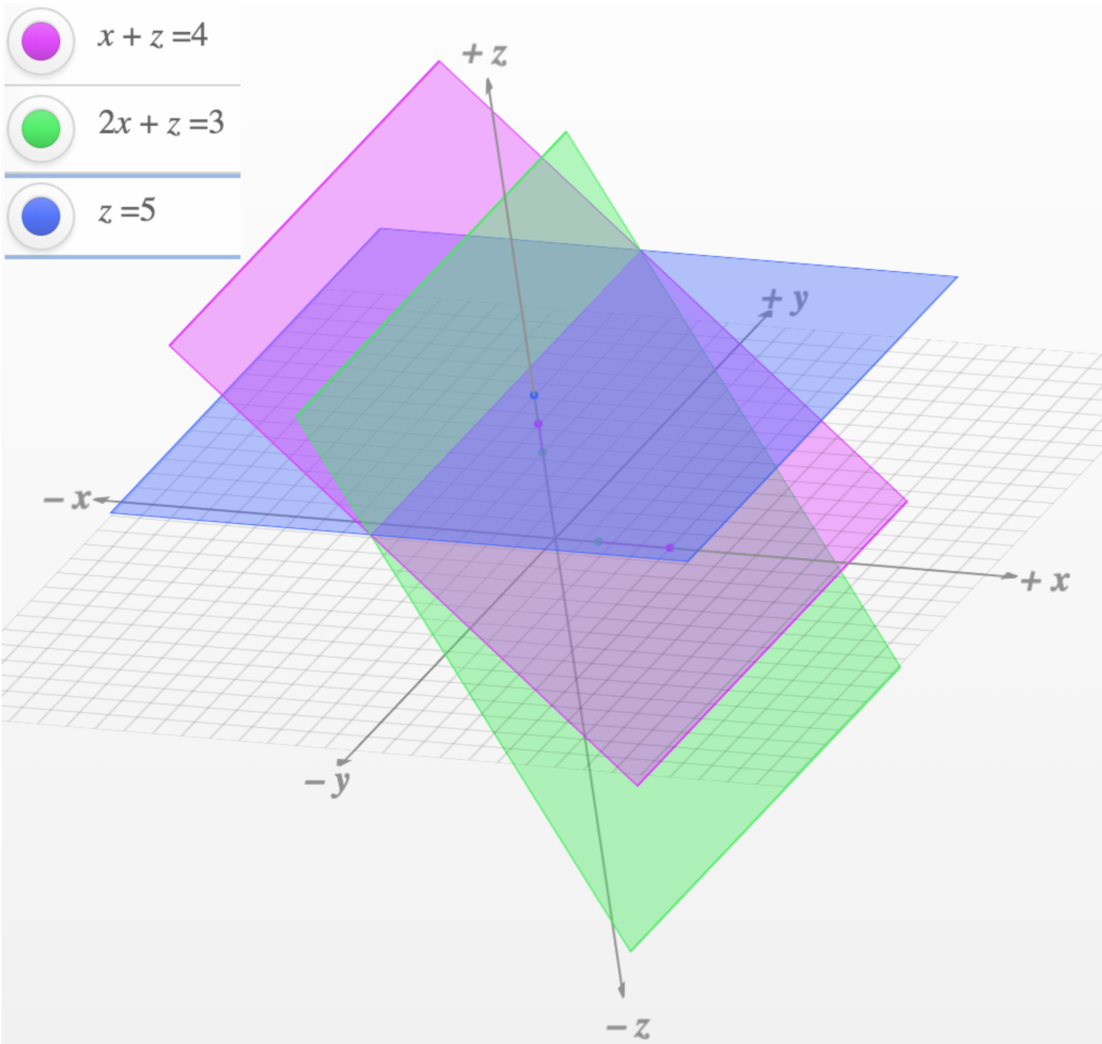
Question 3

The three dimensional graph below is of the three planes,

In purple : $x + z = 4$

In green : $2x + z = 3$

In blue : $z = 5$



The three planes have a common line of intersection.

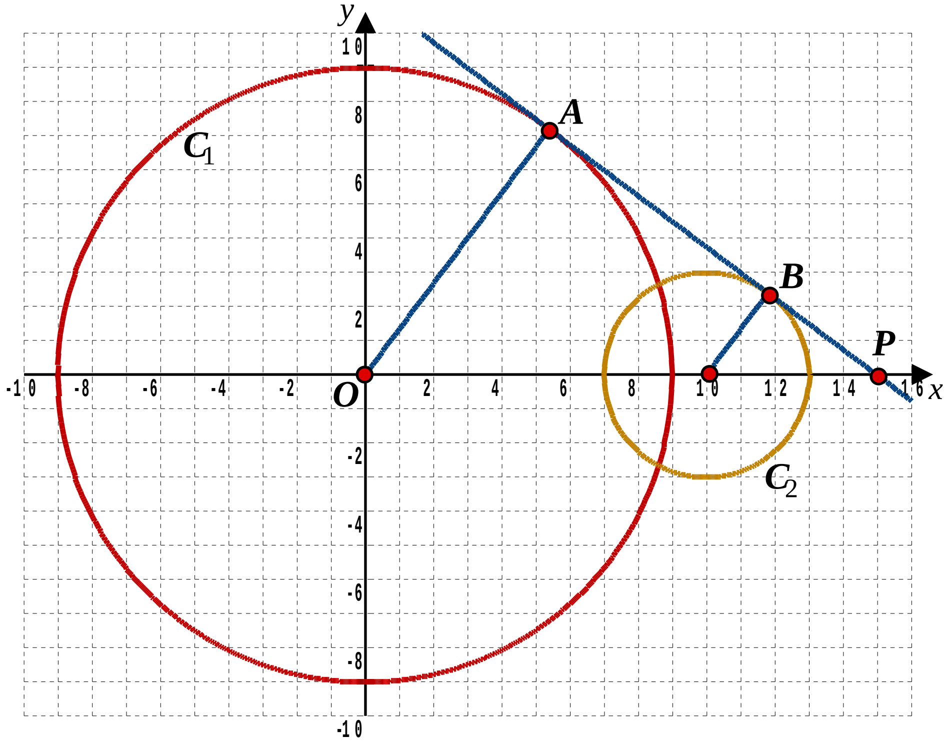
(i) What is this type of configuration of planes called ?

[1 mark]

(ii) Find a vector equation of the line of intersection in the form $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b}$

[3 marks]

Question 4



The circle C_1 has equation $x^2 + y^2 = 81$

The circle C_2 has centre $(10, \ 0)$ and radius 3

(a) Write down the equation of C_2

[1 mark]

The line ABP is a tangent to C_1 at A and is also a tangent to C_2 at B

It cuts the x-axis at the point P

(b) By considering similar triangles, show that the coordinates of P are $(15, 0)$

[3 marks]

(c) A line through P has gradient m.
Write down, in terms of m, the equation of this line.

[1 mark]

This line cuts C_1 in two points.

(d) Show that the x-coordinates of these two points satisfy the equation,

$$x^2 \left(1 + m^2 \right) - 30\, m^2 x + \left(225\, m^2 - 81 \right) = 0$$

[3 marks]

(e) **Hence** determine the coordinates of the point A

[5 marks]

Question 5

By solving a suitable matrix equation with the aid of your calculator, find the single point at which the following three planes intersect,

$$\begin{aligned} x - 3y - 4z &= 3 \\ 6x + 5y - 7z &= 30 \\ x + 4y + 6z &= -3 \end{aligned}$$

[4 marks]