



Doctor, doctor, I'm scared of Christmas.  
Hmmm, I think you're suffering from Claus-trophobia.

*Any solution based entirely on graphical  
or numerical methods is not acceptable*

Marks Available : 40

**Question 1**

Given that  $f(x) = \ln \cos x$

( i ) show that  $f'(x) = -\tan x$

[ 1 mark ]

( ii ) find the values of  $f'(0)$ ,  $f''(0)$ ,  $f'''(0)$  and  $f''''(0)$

[ 4 marks ]

( iii ) express  $\ln \cos x$  as a series in ascending powers of  $x$  up to the term in  $x^4$

[ 2 marks ]

**Question 2**

Three planes  $A$ ,  $B$  and  $C$  are defined by the following equations,

$$A : x + ay + 2z = a$$

$$B : x - y - z = a$$

$$C : x + 4y + 4z = 0$$

Given that the planes do not meet at a single point,

( a ) find the value of  $a$

[ 4 marks ]

( b ) determine whether the three equations form a consistent system, and give a geometric interpretation of your answer.

[ 4 marks ]

**Question 3**

$$f(n) = 2^n + 6^n$$

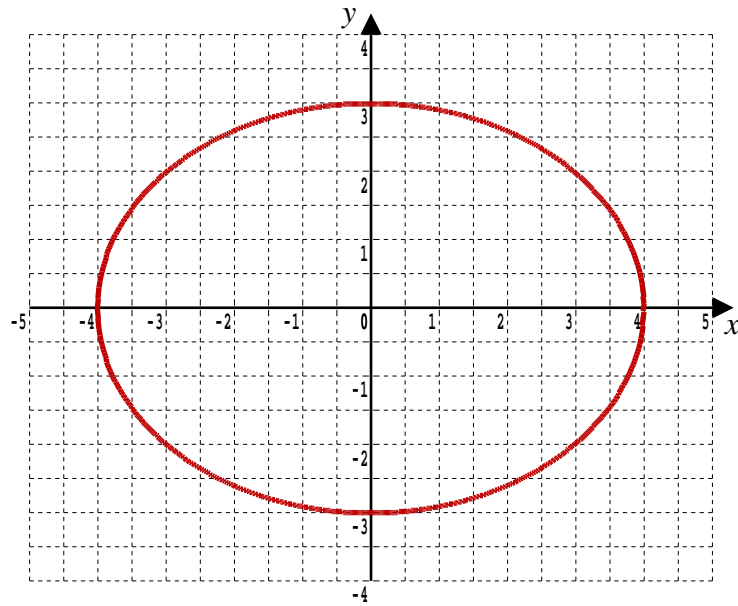
(a) Show that  $f(k + 1) = 6f(k) - 4(2^k)$

[ 3 marks ]

(b) Prove by induction that that all  $n \in \mathbb{Z}^+$   $f(n)$  is divisible by 8

[ 4 marks ]

**Question 4**



An ellipse has parametric equations

$$x = 4 \cos \theta, \quad y = 3 \sin \theta, \quad 0 \leq \theta \leq 2\pi$$

(i) Find the area enclosed by the ellipse.

[ 5 marks ]

(ii) Find the volume of the solid of revolution formed when this area is rotated through  $2\pi$  radians about the  $x$ -axis.

[ 5 marks ]

**Question 5**

The complex number  $w$  satisfies,

$$|w - 1 - i| = 3 \quad \text{and} \quad \arg(w - 2) = \frac{\pi}{4}$$

Find, in simplest form, the exact value of  $|w|^2$

[ 8 marks ]

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In October 2020, Shrewsbury School was voted "**Independent School of the Year 2020**"

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Teachers may obtain detailed worked solutions to the exercises by email from [mhh@shrewsbury.org.uk](mailto:mhh@shrewsbury.org.uk)