Year FM Further Pure Mathematics Examination Revision : Health Check N° 7

Why did Count Dracula go to the doctor ?
He couldn't stop coffin !

Any solution based entirely on graphical
or numerical methods is not acceptable
Marks Available : 42

Question 1

FM A-Level Examination Question from November 2021, Paper Core 2, Q1 (OCR)

Two matrices, **A** and **B** are given by,

$$\mathbf{A} = \begin{pmatrix} 1 & -2 & -1 \\ 2 & -3 & 1 \\ a & 1 & 1 \end{pmatrix} \text{ and } \mathbf{B} = \begin{pmatrix} -6 & 3 & -4 \\ -1 & 6 & -4 \\ 8 & -8 & -1 \end{pmatrix} \text{ where } a \text{ is a constant.}$$

Find the value of a for which $\mathbf{AB} = \mathbf{BA}$

[3 marks]

Question 2

FM A-Level Examination Question from June 2021, Paper 2, Q5 (AQA)

The equation $z^3 + 2z^2 - 5z - 3 = 0$ has roots α, β and γ

Find a cubic with roots $\frac{1}{2}\alpha - 1$, $\frac{1}{2}\beta - 1$ and $\frac{1}{2}\gamma - 1$

[5 marks]

Question 3

FM A-Level Examination Question from June 2017, Paper FP2, Q4 (Edexcel)

$$y = ln\left(\frac{1}{1-2x}\right), \qquad |x| < \frac{1}{2}$$

(a) Find $\frac{dy}{dx}$, $\frac{d^2y}{dx^2}$ and $\frac{d^3y}{dx^3}$

[4 marks]

(b) Hence, or otherwise, find the series expansion of $ln\left(\frac{1}{1-2x}\right)$ about $x = 0$, in ascending powers of x, up to and including the term in x^3. Give each coefficient in its simplest form.

[3 marks]

(c) Use your expansion to find an approximate value for $ln\left(\frac{3}{2}\right)$ giving your answer to 3 decimal places.

[3 marks]

Question 4

FM AS-Level Examination Question from May 2019, Paper Core, Q4 (OCR)

In this question you must show detailed reasoning.

You are given that $f(z) = 4z^4 - 12z^3 + 41z^2 - 128z + 185$ and that $2 + i$ is a root of the equation $f(z) = 0$

(a) Express $f(z)$ as the product of two quadratic factors with integer coefficiants.

[5 marks]

(b) Solve $f(z) = 0$

[3 marks]

Two loci on an Argand diagram are defined by

$C_1 = \{ z : |z| = r_1 \}$ and $C_2 = \{z : |z| = r_2\}$ where $r_1 > r_2$

You are given that two of the points representing the roots of $f(z) = 0$ (which you worked out in part (b)) are on C_1 and two are on C_2

Let R be the region on the Argand diagram between C_1 and C_2

(c) Find the exact area of R

[4 marks]

(d) ω is the sum of all the roots of $f(z) = 0$

Determine whether or not the point on the Argand diagram which represents ω lies in R.

[2 marks]

Question 5

Further Mathematics Examination Question from January 2012, Q7 (ii) (OCR)

It is given that x satisfies the equation $arsinh\, x - arcosh\, x = ln\, 2$

(i) Use the logarithmic forms for $arsinh\, x$ and $arcosh\, x$ to show that,

$$\sqrt{x^2 + 1} - 2\sqrt{x^2 - 1} = x$$

[2 marks]

(ii) Hence, by squaring this equation, find the exact value of x

[3 marks]

Question 6

FM AS-Level Examination Question from May 2019, Paper Core, Q6 (OCR)

A transformation T is represented by the matrix $\mathbf{T}$ where,

$$\mathbf{T} = \begin{pmatrix} x^2 + 1 & -4 \\ 3 - 2x^2 & x^2 + 5 \end{pmatrix}$$

A quadrilateral Q, whose area is 12 units, is transformed by T to Q'

Find the smallest possible value of the area of Q'

[5 marks]