Year FM Further Pure Mathematics Examination Revision : Health Check N° 8

“Doctor, doctor, I'm addicted to brake fluid”
“What nonsense, you can stop anytime”

Any solution based entirely on graphical
or numerical methods is not acceptable

Marks Available : 40

Question 1

Given a complex number $z = a + b\text{i}$, the conjugate of z, which is denoted z^*, is the complex number $z = a - b\text{i}$.

Show that $\dfrac{z}{z^*} = \left(\dfrac{a^2 - b^2}{a^2 + b^2} \right) + \left(\dfrac{2ab}{a^2 + b^2} \right)\text{i}$

[3 marks]

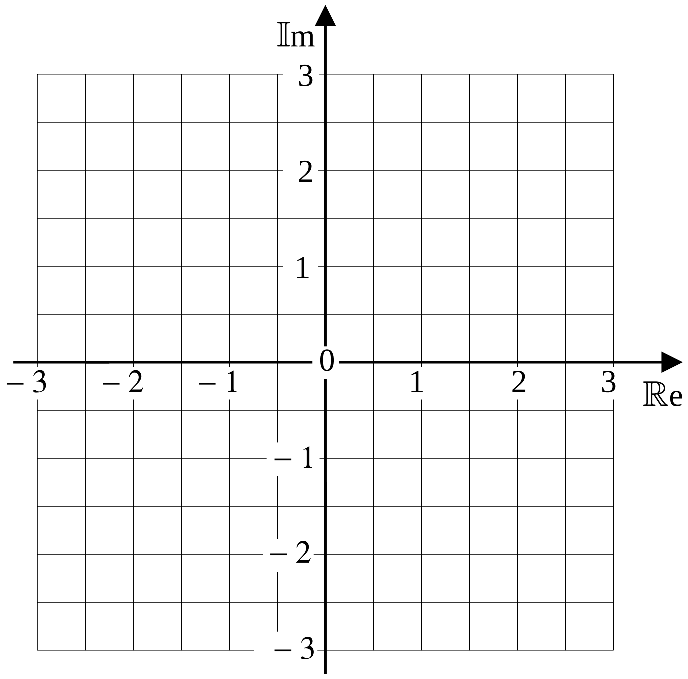
Question 2

FM A-Level Examination Question from June 2021, Paper 1, Q6 (AQA)

(a) Show that the equation $\left(2z - z^*\right)^* = z^2$ has exactly four solutions.
Find these solutions.

[7 marks]

(b) **(i)** Plot the four solutions to the equation in part (a) on the Argand diagram and join them together to form a quadrilateral with one line of symmetry.



[2 marks]

(ii) Show that the area of this quadrilateral is $\dfrac{\sqrt{15}}{2}$ square units.

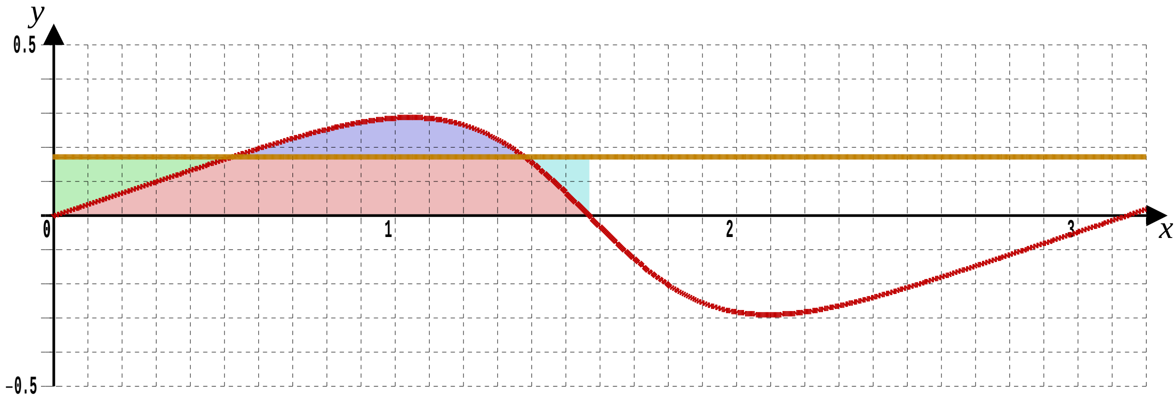
[1 mark]

Question 3

(a) Find the exact mean value of $f(x) = \dfrac{\sin x \cos x}{\cos 2x + 2}$ over the interval $\left[0, \dfrac{\pi}{2}\right]$

[4 marks]

(b) The graph is of the function $f(x) = \dfrac{\sin x \cos x}{\cos 2x + 2}$



Making use of the graph, explain the geometric significance of your part (a) answer.

[2 marks]

Question 4

FM A-Level Examination Question from October 2021, Paper Core 1, Q1 (OCR)

(a) Sketch on a single Argand diagram the loci given by,

(i) $|z - 1 + 2\mathrm{i}| = 3$

[2 marks]

(ii) $|z + 1| = |z - 2|$

[2 marks]

(b) Indicate, by shading, the region of the Argand diagram for which

$$|z - 1 + 2\mathrm{i}| \leqslant 3 \text{ and } |z + 1| \leqslant |z - 2|$$

[2 marks]

Question 5

(a) Use the substitution $x = \dfrac{a}{\sinh \theta}$, where a is a constant, to show that,

for $x > 0, a > 0,$ $\displaystyle\int \frac{1}{x\sqrt{x^2 + a^2}}\,dx = -\frac{1}{a}\,\text{arsinh}\left(\frac{a}{x}\right) + \text{constant}$

[6 marks]

(b) Hence, or otherwise, find the exact value of $\displaystyle\int_1^2 \frac{1}{x\sqrt{x^2 + 4}}\,dx$

[4 marks]

Question 6

The Cartesian equation of a curve is $\left(x^2 + y^2 - 2x\right)^2 = 4\left(x^2 + y^2\right)$

Recast this equation in the polar form, $r = f(\theta)$

[5 marks]