Year FM Further Pure Mathematics Examination Revision : Health Check N° 9

"Doctor, doctor, Will this ointment clear my spots ?"
"I never make rash promises"

Any solution based entirely on graphical
or numerical methods is not acceptable

Marks Available : 40

Question 1

FM A-Level Examination Question from October 2021, Paper Core Pure, Q2 (MEI)

Find the gradient of the curve $y = 6\,arcsin\,(2x)$ at the point with x-coordinate $\frac{1}{4}$

Express the result in the form $m\sqrt{n}$, where m and n are integers.

[4 marks]

Question 2

FM A-Level Examination Question from October 2021, Paper Core Pure, Q14 (MEI)

A curve has polar equation $r = a(\cos\theta + 2\sin\theta)$, where a is a positive constant and $0 \leqslant \theta \leqslant \pi$

(a) Determine the polar coordinates of the point on the curve which is furthest from the pole.

[7 marks]

(b) **(i)** Show that the curve is a circle whose radius should be specified.

[6 marks]

(ii) Write down the polar coordinates of the centre of the circle.

[1 mark]

Question 3

(i) Explain what it means for an integral to be improper.

[1 mark]

(ii) Identify two features of $\int_0^{\infty} \frac{1}{(x+1)\sqrt{x}}\,dx$ which make it improper.

[1 mark]

(iii) By differentiating $arctan\sqrt{x}$, or otherwise, show that

$\int_0^{\infty} \frac{1}{(x+1)\sqrt{x}}\,dx$ is convergent and find its exact value.

[5 marks]

Question 4

FM A-Level Question from October 2020, Paper Core Pure 1, Q9 (OCR)

You are given that the cubic equation $2x^3 + px^2 + qx - 3 = 0$, where p and q are real numbers, has a complex root $\alpha = 1 + i\sqrt{2}$

(a) Write down a second complex root, β

[1 mark]

(b) Determine the third root, γ

[2 marks]

(c) Find the value of p and the value of q

[2 marks]

(d) Show that if n is an integer then

$$\alpha^n + \beta^n + \gamma^n = 2 \times 3^{\frac{n}{2}} \times \cos n\theta + \frac{1}{2^n} \quad \text{where } \tan\theta = \sqrt{2}$$

[4 marks]

Question 5

FM AS-Level Examination Question from October 2020, Paper Pure Core, Q1 (OCR)

In this question you must show detailed reasoning.

Use an algebraic method to find the square roots of $(-77 - 36\mathrm{i})$

[6 marks]