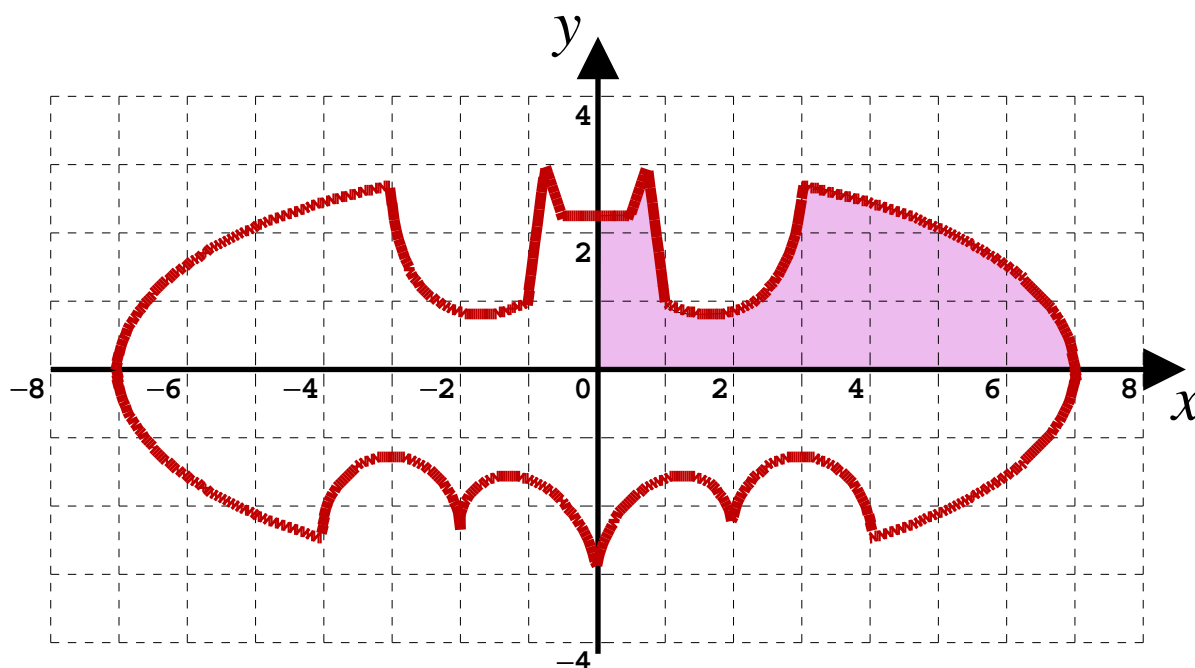


A-Level Pure Mathematics  
Year 2

# INTEGRATION

## III



“The Batman Curve”

$$\text{Four curves base} : f(x) = \left| \frac{x}{2} \right| - \frac{3\sqrt{33} - 7}{122} x^2 - 3 + \sqrt{1 - (||x| - 2| - 1)^2}$$

$$\text{Sides of face} : g(x) = 9\sqrt{\frac{|1 - (|x| - 1)(|x| - 0.75)|}{-(|x| - 1)(|x| - 0.75)}} - 8|x|$$

$$\text{Ears} : h(x) = 3|x| + 0.75\sqrt{\frac{|1 - (|x| - 0.5)(|x| - 0.75)|}{-(|x| - 0.5)(|x| - 0.75)}}$$

$$\text{Forehead} : p(x) = 2.25\sqrt{\frac{|1 - (x - 0.5)(x + 0.5)|}{-(x - 0.5)(x + 0.5)}}$$

$$\text{Shoulders} : q(x) = \frac{6\sqrt{10}}{7} + (1.5 - 0.5|x|)\sqrt{\frac{||x| - 1|}{|x| - 1}} - \frac{6\sqrt{10}}{14}\sqrt{4 - (|x| - 1)^2}$$

$$\text{Wings} : r(x) = 3\sqrt{1 - \left(\frac{x}{7}\right)^2}\sqrt{\frac{||x| - 3|}{|x| - 3}} \quad s(x) = -3\sqrt{1 - \left(\frac{x}{7}\right)^2}\sqrt{\frac{||x| - 4|}{|x| - 4}}$$

## Lesson 1

## A-Level Pure Mathematics : Year 2 Integration III

### 1.1 Integration by Substitution

#### Example

*A-Level Examination Question from January 2006, Paper C4, Q3 (Edexcel)*

Using the substitution  $u^2 = 2x - 1$ , or otherwise, find the exact value of

$$\int_1^5 \frac{3x}{\sqrt{2x-1}} dx$$

[ 8 marks ]

## 1.2 Exercise

*Any solution based entirely on graphical or numerical methods is not acceptable*

Marks Available : 50

### Question 1

Evaluate the following integral using the substitution given;

$$\int_3^{11} \frac{x}{\sqrt{x-2}} dx \quad u^2 = x - 2$$

[ 8 marks ]

**Question 2**

Evaluate the following integral using the substitution given;

$$\int_6^{10} \frac{x(x-4)}{(x-2)^2} dx \quad u = x - 2$$

[ 8 marks ]

**Question 3**

Evaluate exactly the following integral using the substitution given;

$$\int_2^6 \frac{x-1}{\sqrt{2x-3}} dx \quad u^2 = 2x-3$$

[ 8 marks ]

**Question 4**

Evaluate the following integral using the substitution given;

$$\int_{0.5}^1 \frac{1}{\sqrt{1-x^2}} dx \quad x = \sin \theta$$

**Warning :** You must use **RADIANS**

[ 8 marks ]

**Question 5**

Evaluate the following integral using the substitution given;

$$\int_2^{2.5} \frac{2x + 1}{(3 - x)^6} dx \quad u = 3 - x$$

[ 9 marks ]

**Question 6**

Evaluate the following integral using the substitution given;

$$\int_{-0.25}^{0.5} \frac{48x}{(2x + 1)^5} dx \quad u = 2x + 1$$

[ 9 marks ]

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In October 2020, Shrewsbury School was voted "**Independent School of the Year 2020**"

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Teachers may obtain detailed worked solutions to the exercises by email from [mhh@shrewsbury.org.uk](mailto:mhh@shrewsbury.org.uk)