3.1 Tricky Integration by Parts

$f(x)$	$f'(x)$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$
$\sec x$	$\sec x \ \tan x$
$\csc x$	$-\csc x \ \cot x$
$\cot x$	$-\csc^2 x$
$\ln x$	$\frac{1}{x}$
e^x	e^x

Using the table of derivatives from right to left we can see that, for example;

$$\int \sin x \ dx \ = \ -\cos x \ + \ c$$

$$\int \cos x \ dx \ = \ \sin x \ + \ c$$

However, there are some obvious omissions.
For example;

$$\int \tan x \ dx$$

Some cunning is needed to find this integral.

$$\int \tan x \ dx \ = \ \int \frac{\sin x}{\cos x} \ dx$$

$$= \ \int (\sin x)(\cos x)^{-1} \ dx$$

$$= \ (-1) \int (-\sin x)(\cos x)^{-1} \ dx$$

$$= \ (-1) \ \ln | \ \cos x \ | \ + c$$

$$= \ \ln \ | \cos x \ |^{-1} \ + \ c$$

$$= \ \ln \left| \frac{1}{\cos x} \right| \ + \ c$$

$$= \ \ln | \ \sec x \ | \ \ + \ c$$

[4 marks]

3.2 Exercise

*Any solution based entirely on graphical
or numerical methods is not acceptable*

Marks Available : 50

Question 1

Use the result, just proved, to find show that;

$$\int_0^{\frac{\pi}{6}} \tan x \ dx \ = \ \ln 2 \ - \ \frac{1}{2} \ln 3$$

[4 marks]

Question 2

Use integration by parts to find;

$$\int x \ \sec^2 x \ dx$$

[4 marks]

Question 3

(**a**)   Use the fact that;

$$cos^2 x + sin^2 x = 1$$

to prove that;

$$tan^2 x = sec^2 x - 1$$

[**2 marks**]

(**b**)   Hence, or otherwise, find;

$$\int x \, tan^2 x \, dx$$

[**3 marks**]

Question 4

Mirror the "cunning" used to integrate *tan x* to find an expression for;

$$\int cot x \, dx$$

[**4 marks**]

Question 5

Use your question 4 result to find the *exact* value of;

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \cot x \ dx$$

[4 marks]

Question 6

Use integration by parts to find;

$$\int x \ \csc^2 x \ dx$$

[5 marks]

Question 7

Using a trigonometric formula and integration by parts, or otherwise, find;

$$\int x \cot^2 x \ dx$$

[5 marks]

Question 8

Using a trigonometric formulae first, or otherwise, find;

$$\int \cos^2 x \ dx$$

[5 marks]

Question 9
Use integration by parts, and your question 8 result, to find;

$$\int x \cos^2 x \; dx$$

[7 marks]

Question 10
Use integration by parts to help find;

$$\int \frac{\ln x}{x} \; dx$$

[7 marks]