Lesson 4

A-Level Pure Mathematics : Year 2

Integration III

4.1 Examination Questions on Integration by Parts

$f(x)$	$f'(x)$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$
$\sec x$	$\sec x \tan x$
$\csc x$	$-\csc x \cot x$
$\cot x$	$-\csc^2 x$
$\ln x$	$\frac{1}{x}$
$\ln \lvert \sec x \rvert$	$\tan x$
$\ln \lvert \sin x \rvert$	$\cot x$
e^x	e^x

4.2 Exercise

Any solution based entirely on graphical or numerical methods is not acceptable

Marks Available : 47

Question 1

A-Level Examination Question from January 2011, Paper C4, Q1 (Edexcel)

Use integration by parts to find the exact value of

$$\int_0^{\frac{\pi}{2}} x \sin 2x \, dx$$

[6 marks]

Question 2

A-Level Examination Question from January 2012, Paper C4, Q2 (Edexcel)

(a) Use integration by parts to find

$$\int x \sin 3x \, dx$$

[3 marks]

(b) Using your answer to part (a), find

$$\int x^2 \cos 3x \, dx$$

[3 marks]

Question 3

A-Level Examination Question from June 2007, Paper C4, Q3 (Edexcel)

(a) Find

$$\int x \cos 2x \, dx$$

[4 marks]

(b) Hence, using the identity

$$\cos 2x = 2\cos^2 x - 1$$

deduce

$$\int x \cos^2 x \, dx$$

[3 marks]

Question 4

A-Level Examination Question from January 2008, Paper C4, Q4 (Edexcel)

(i) Find

$$\int ln\left(\frac{x}{2}\right) dx$$

[4 marks]

(ii) Find the exact value of

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} sin^2 x \ dx$$

[5 marks]

Question 5

A-Level Examination Question from June 2008, Paper C4, Q2 (Edexcel)

(a) Use integration by parts to find

$$\int x\, e^{x}\ dx$$

[3 marks]

(b) Hence find

$$\int x^{2}\, e^{x}\ dx$$

[3 marks]

Question 6

A-Level Examination Question from January 2009, Paper C4, Q6 (Edexcel)

(a) Find

$$\int tan^2 x \ dx$$

[2 marks]

(b) Use integration by parts to find

$$\int \frac{1}{x^3} ln\, x \ dx$$

[4 marks]

(c) Use the substitution $u = 1 + e^x$ to show that,

$$\int \frac{e^{3x}}{1 + e^x}\, dx = \frac{1}{2} e^{2x} - e^x + ln\,(1 + e^x) + k$$

where k is a constant.

[7 marks]