5.1 Year 2 Integration : Examination Questions

The formulae book provided in the examination gives the derivative of many functions. These are identified with a * in the table below, which also highlights several key results that are NOT provided. Used backward, the table gives integrals.

$f(x)$	$f'(x)$	Given ?
$\sin x$	$\cos x$	
$\cos x$	$-\sin x$	
$\tan x$	$\sec^2 x$	*
$\sec x$	$\sec x \tan x$	*
$\csc x$	$-\csc x \cot x$	*
$\cot x$	$-\csc^2 x$	*
$\ln x$	$\frac{1}{x}$	
$\ln\lvert\sec x\rvert$	$\tan x$	*
$\ln\lvert\sin x\rvert$	$\cot x$	*
$\ln\lvert\sec x + \tan x\rvert$	$\sec x$	*
$\ln\left\lvert\tan\left(\frac{1}{2}x + \frac{1}{4}\pi\right)\right\rvert$	$\sec x$	*
$-\ln\lvert\csc x + \cot x\rvert$	$\csc x$	*
$\ln\left\lvert\tan\left(\frac{1}{2}x\right)\right\rvert$	$\csc x$	*
e^x	e^x	

As has been seen, many questions require the application of a trigonometric identity, but the useful identities are not given explicitly.

The ***three key*** identities should be memorised;

$$\cos^2\theta + \sin^2\theta = 1$$

$$\cos^2\theta - \sin^2\theta = \cos 2\theta$$

$$2\sin\theta\cos\theta = \sin 2\theta$$

From the ***three key***, the ***following four*** are easily obtained;

$$1 + \tan^2\theta = \sec^2\theta$$

$$\cot^2\theta + 1 = \csc^2\theta$$

$$2\cos^2\theta = 1 + \cos 2\theta \qquad \text{Essential to find } \int \cos^2\theta\, d\theta$$

$$2\sin^2\theta = 1 - \cos 2\theta \qquad \text{Essential to find } \int \sin^2\theta\, d\theta$$

Either memorise the ***following four***, or learn how to obtain them from the ***three key***.

5.2 Exercise

Any solution based entirely on graphical
or numerical methods is not acceptable
Marks Available : 28

When trigonometry and calculus mix, **RADIANS MUST BE USED !**

Question 1

A-Level Examination Question from January 2010, Paper C4, Q8 (a) (Edexcel)

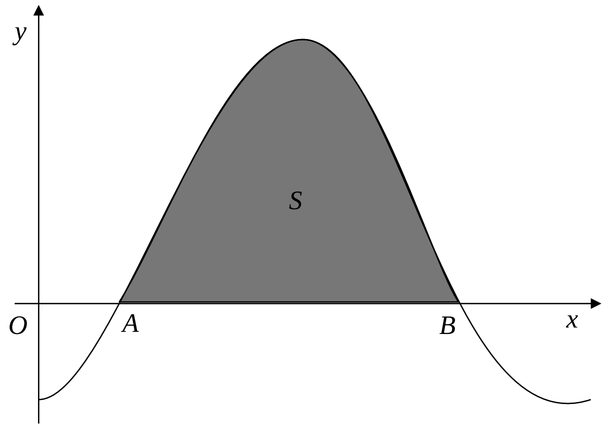
Using the substitution $x = 2 \cos u$, or otherwise, find the exact value of

$$\int_{1}^{\sqrt{2}} \frac{1}{x^2 \sqrt{4 - x^2}} \, dx$$

[7 marks]

Question 2

A-Level Examination Question from January 2013, Paper C4, Q6 (a) (Edexcel)



Shown is a sketch of the curve with equation $y = 1 - 2\cos x$, where x is measured in radians. The curve crosses the x-axis at the point A and the point B.

Find, in terms of π, the x coordinate of the point A and the x coordinate of the point B.

[3 marks]

Question 3

A-Level Examination Question from June 2013, Paper C4, Q5 (Edexcel)

(a) Use the substitution $x = u^2$, $u > 0$, to show that,

$$\int \frac{1}{x\,(2\sqrt{x} - 1)}\, dx = \int \frac{2}{u\,(2u - 1)}\, du$$

[3 marks]

(b) Hence show that

$$\int_1^9 \frac{1}{x\,(2\sqrt{x} - 1)}\, dx = 2\, ln \left(\frac{a}{b}\right)$$

where a and b are integers to be determined.

[7 marks]

Question 4

A-Level Examination Question from June 2004, Paper P3, Q4 (Edexcel)

Use the substitution $u = 1 + \sin x$ and integration to show that

$$\int \sin x \cos x \,(1 + \sin x)^5 \, dx = \frac{1}{42}(1 + \sin x)^6 \,(6 \sin x - 1) + constant$$

[8 marks]

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