Lesson 6

A-Level Pure Mathematics : Year 2
Integration III

6.1 Parametric Integration

Example

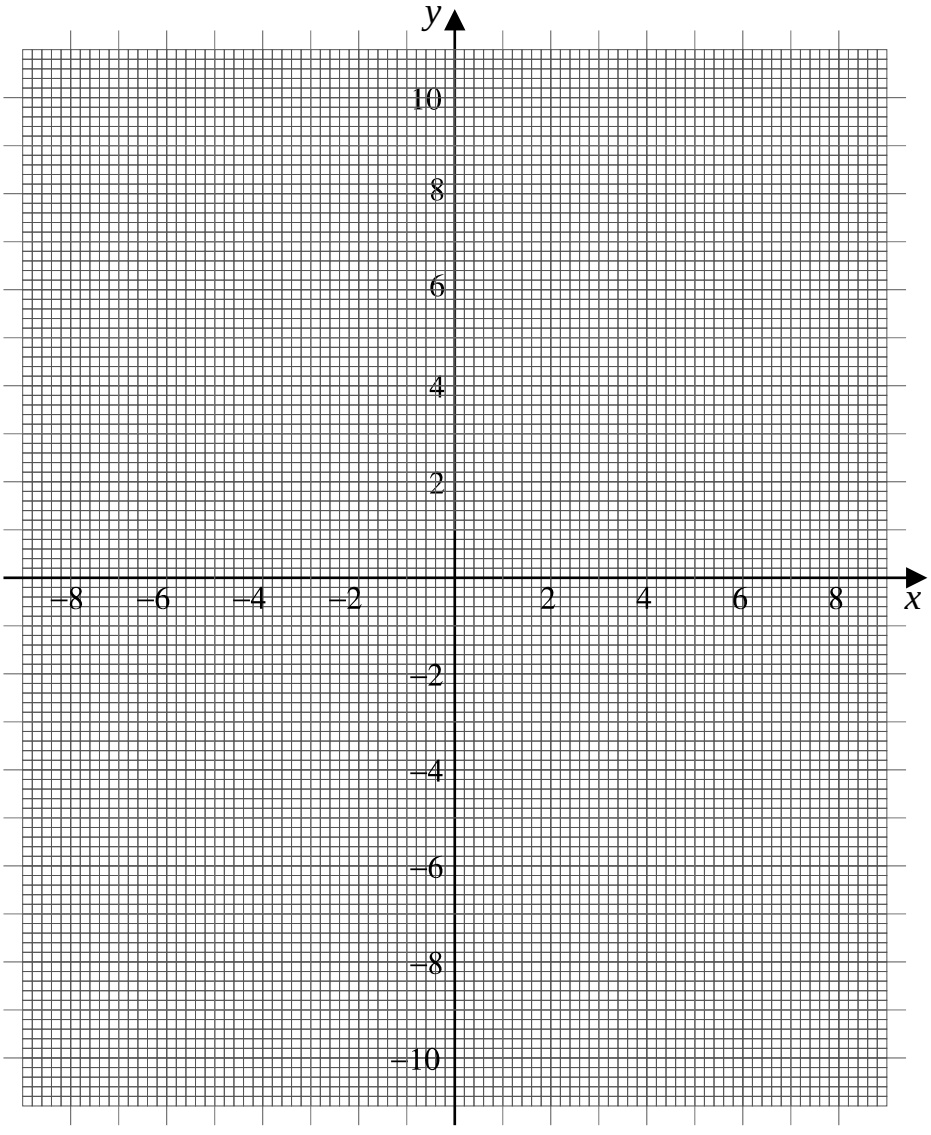
(i) Complete the following table to determine some points on the curve with parametric equations,

$$x = t^2 - 9 \qquad y = 2t$$

t	-4	-3	-2	-1	0	1	2	3	4
x									
y									

[3 marks]

(ii) Plot the curve;



[3 marks]

(iii) Shade in the region bounded by the y-axis and the curve.
This is the area to be found by parametric integration.

[1 mark]

(iv) Find the area of the region bounded by the y-axis and the curve with parametric equations,

$$x = t^2 - 9 \qquad\qquad y = 2t$$

[4 marks]

(v) Eliminate t to obtain the Cartesian equation of the curve.

[2 marks]

(vi) Verify your part (iv) answer by integrating the Cartesian equation of the curve between appropriate limits.

[3 marks]

6.2 Exercise

Any solution based entirely on graphical or numerical methods is not acceptable

Marks Available : 20

Question 1

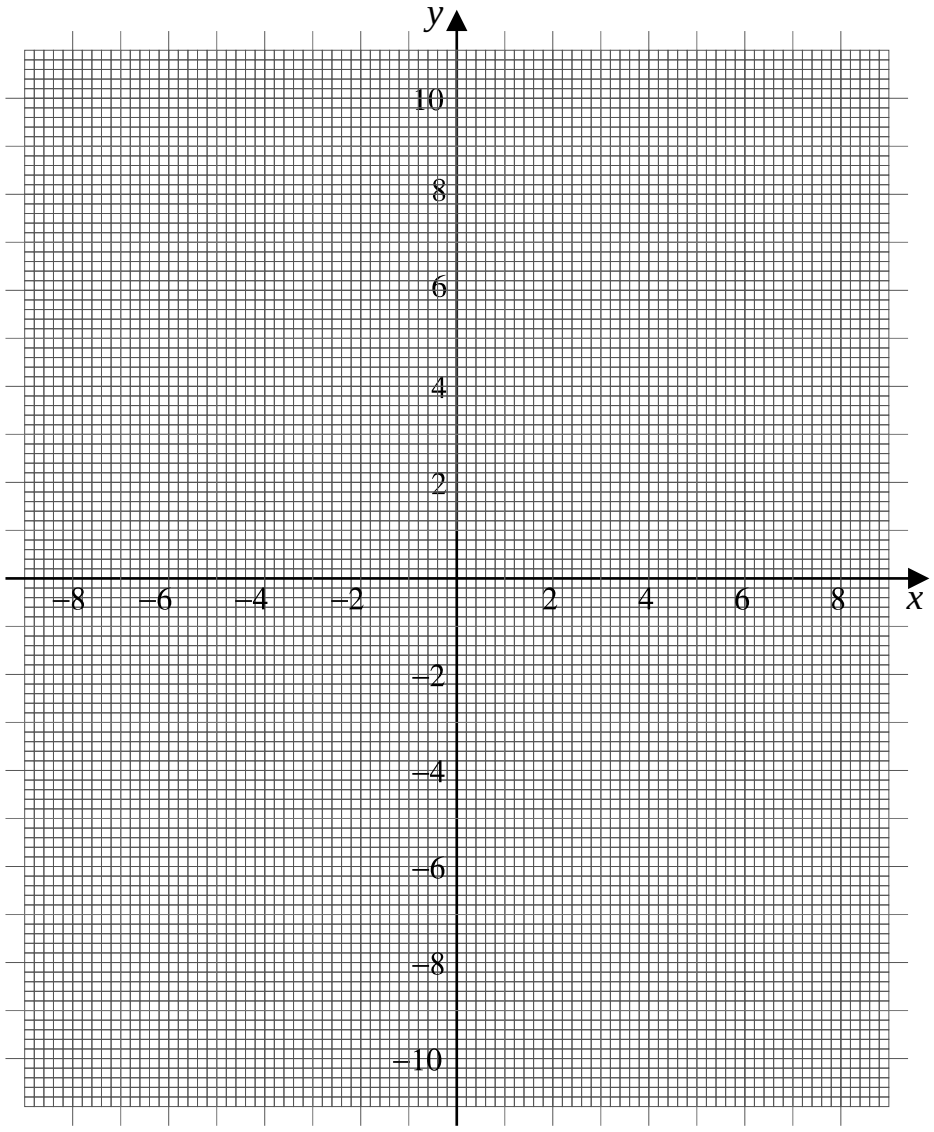
(i) Complete the following table to determine some points on the curve with parametric equations

$$x = 4 - t^2 \qquad y = t\left(t^2 - 9\right)$$

t	– 3.4	– 3	– 2	– 1	0	1	2	3	3.4
x	–7.6								– 7.6
y	– 8.7								8.7

[3 marks]

(ii) Plot the curve;



[3 marks]

(iii) Shade in the region bounded by the loop of the curve.
This is the area to be found by parametric integration.

[1 mark]

(iv) Find the area of the region bounded by the loop of the curve with parametric equations,

$$x = 4 - t^2 \qquad\qquad y = t\left(t^2 - 9\right)$$

[4 marks]

(v) Eliminate t to show that the Cartesian equation of the curve is

$$y = \pm (4 - x)^{\frac{1}{2}} (x + 5)$$

[4 marks]

(vi) Verify your part **(iv)** answer by integrating the Cartesian equation of the curve between appropriate limits.
Use the substitution $u = 4 - x$

[5 marks]