

6.1 Parametric Integration

Example

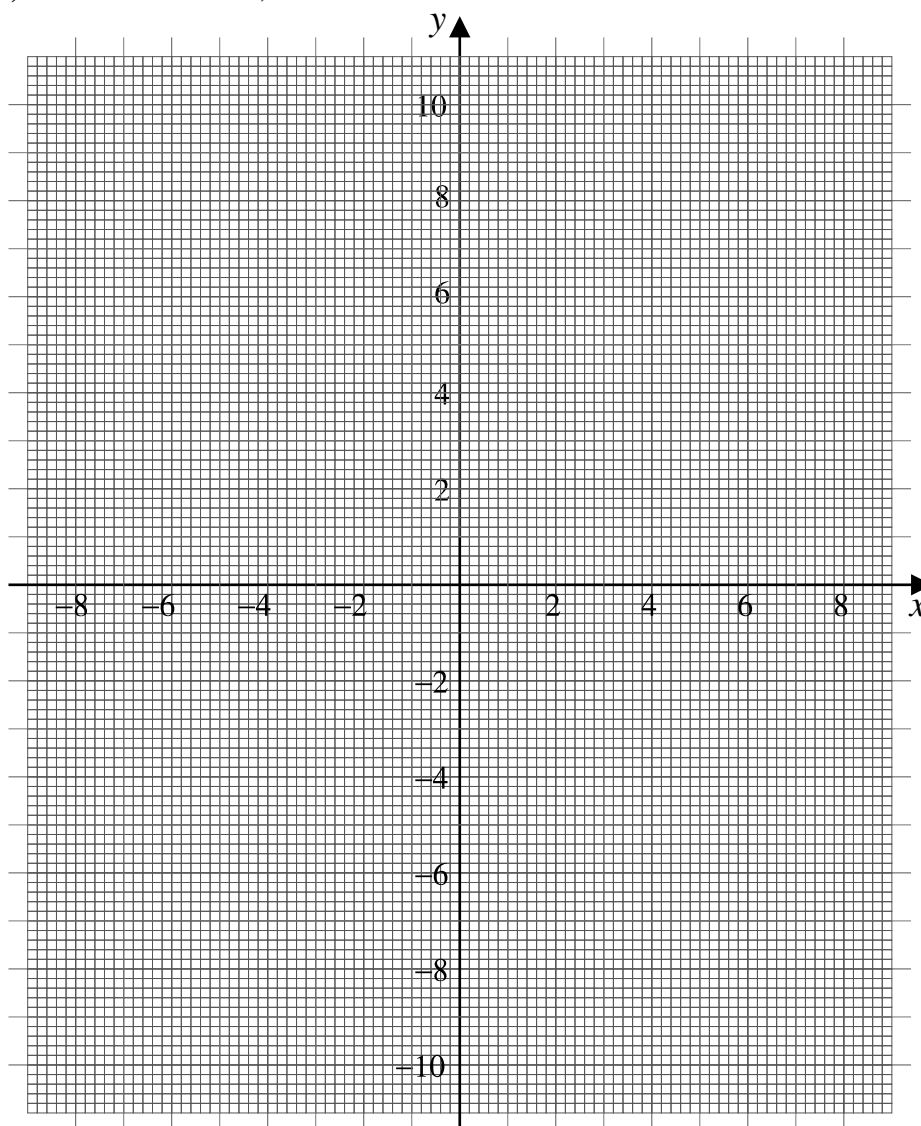
- (i) Complete the following table to determine some points on the curve with parametric equations,

$$x = t^2 - 9 \qquad y = 2t$$

<i>t</i>	-4	-3	-2	-1	0	1	2	3	4
<i>x</i>									
<i>y</i>									

[3 marks]

- (ii) Plot the curve;



[3 marks]

- (iii) Shade in the region bounded by the y -axis and the curve.
This is the area to be found by parametric integration.

[1 mark]

- (iv) Find the area of the region bounded by the y -axis and the curve with parametric equations,

$$x = t^2 - 9 \qquad y = 2t$$

[4 marks]

- (v) Eliminate t to obtain the Cartesian equation of the curve.

[2 marks]

- (vi) Verify your part (iv) answer by integrating the Cartesian equation of the curve between appropriate limits.

[3 marks]

6.2 Exercise

Any solution based entirely on graphical or numerical methods is not acceptable

Marks Available : 20

Question 1

- (i) Complete the following table to determine some points on the curve with parametric equations

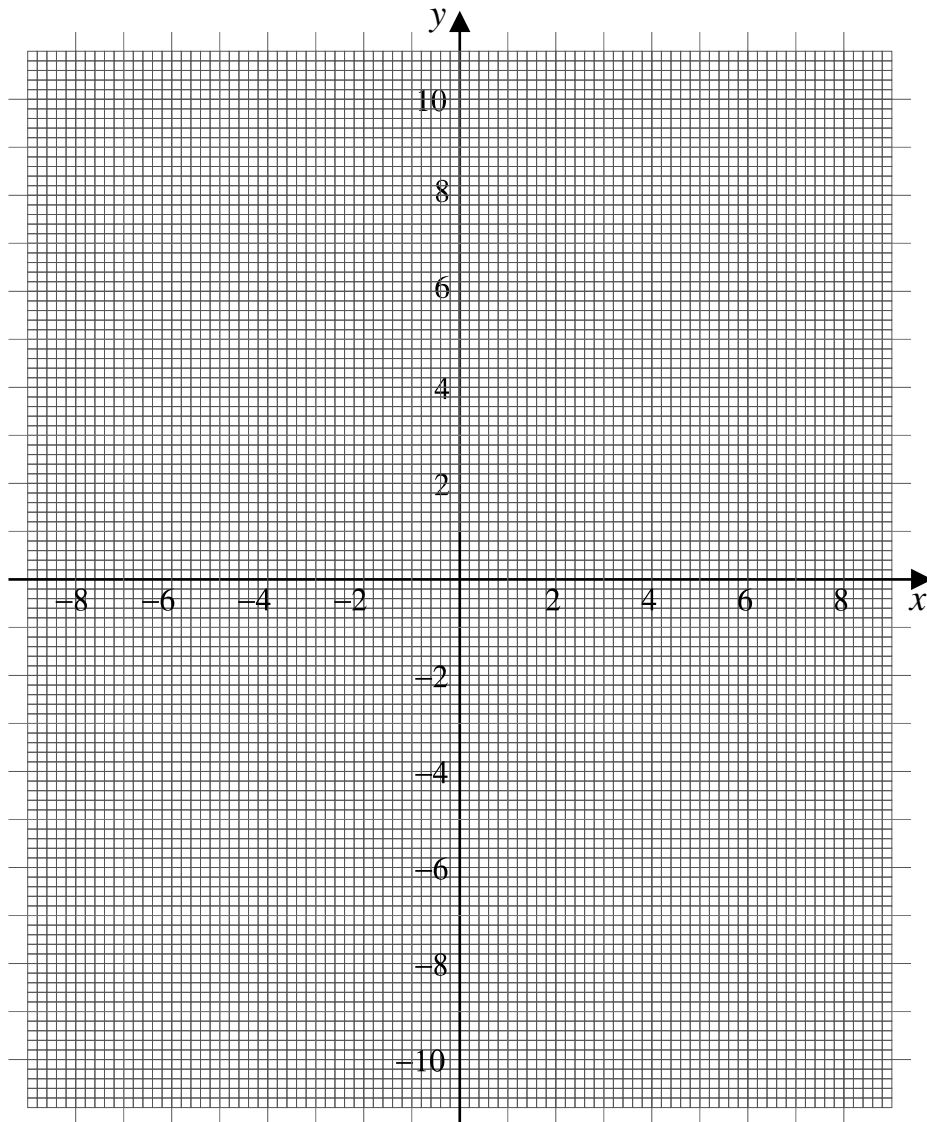
$$x = 4 - t^2$$

$$y = t(t^2 - 9)$$

t	-3.4	-3	-2	-1	0	1	2	3	3.4
x	-7.6								-7.6
y	-8.7								8.7

[3 marks]

- (ii) Plot the curve;



[3 marks]

- (iii) Shade in the region bounded by the loop of the curve.
This is the area to be found by parametric integration.

[1 mark]

- (iv) Find the area of the region bounded by the loop of the curve with parametric equations,

$$x = 4 - t^2 \qquad y = t (t^2 - 9)$$

[4 marks]

- (v) Eliminate t to show that the Cartesian equation of the curve is

$$y = \pm (4 - x)^{\frac{1}{2}} (x + 5)$$

[4 marks]

- (vi) Verify your part (iv) answer by integrating the Cartesian equation of the curve between appropriate limits.
Use the substitution $u = 4 - x$

[5 marks]

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Teachers may obtain detailed worked solutions to the exercises by email from mhh@shrewsbury.org.uk