**Lesson 7**

**A-Level Pure Mathematics : Year 2**

**Integration III**

## 7.1 Sweating the Exam

**Example**

This question is about finding the area in the quadrant between the positive $x$-axis and the positive $y$-axis and the curve, $C$, with parametric equations,

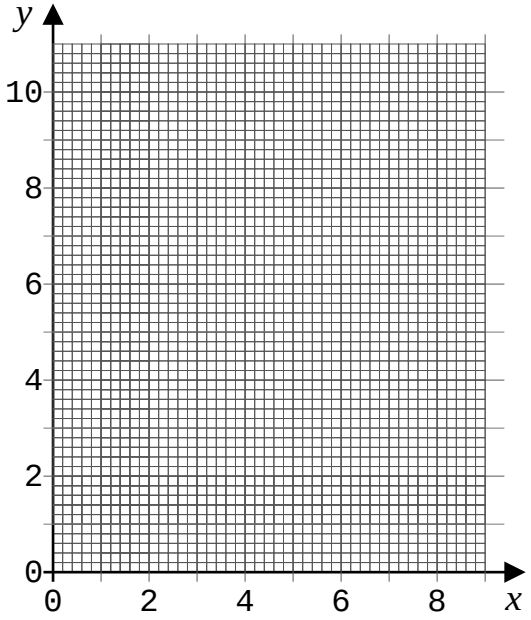
$$x = 4 - t^2 \text{ and } \quad y = t\,(t + 1) \quad \text{for} \quad t \geqslant 0$$

**( i )** Complete the following table,

| $t$ | 0 | 0.5 | 1 | 1.5 | 2 |
|---|---|---|---|---|---|
| $x$ | | | | | |
| $y$ | | | | | |

**[ 2 marks ]**

**( ii )** Use the table to sketch the curve, $C$, and shade the area to be found.




**[ 2 marks ]**

**( iii )** Use parametric integration to find the area.
The following teaching video demonstrates a method of solution, and may be used to help write out your solution, if required.
http://www.NumberWonder.co.uk/Video/v9045(11a).mp4

Notice that, as the bend is anticlockwise as $t$ increases, strictly speaking the integration gives the area sought but with a negative sign. The video has a convincing dodge to argue about which is the lower and which is the upper limit. The following rule is worth keeping in mind;

---

**The Limit Swap Rule**

$$\int_A^B -f(x)\,dx = \int_B^A f(x)\,dx$$

---

The video begins having already done the following step,

$$Area = \int y\,dx = \int y\,\frac{dx}{dt}\,dt$$

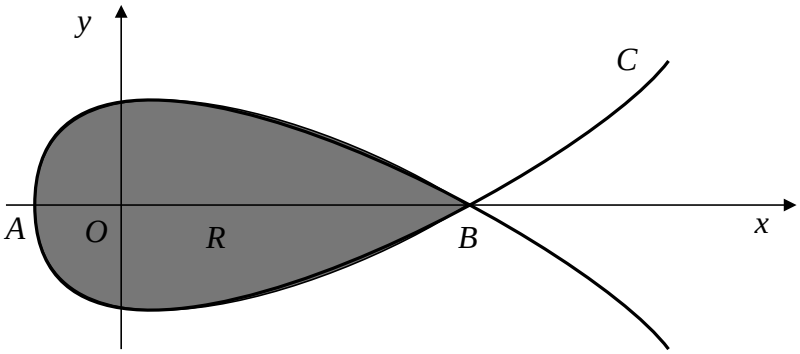
**[ 6 marks ]**

**7.2 Exercise**

*Any solution based entirely on graphical*
*or numerical methods is not acceptable*
Marks Available : 34

**Question 1**

*A-Level Examination Question from January 2010, Paper C4, Q7 (Edexcel)*




The diagram shows a sketch of the curve $C$ with parametric equations

$$x = 5t^2 - 4 \qquad y = t\left(9 - t^2\right)$$

The curve $C$ cuts the $x$-axis at the points $A$ and $B$

**( a )** Find the $x$-coordinate at the point $A$ and the $x$-coordinate at the point $B$

**[ 3 marks ]**

The region $R$, shown shaded, is enclosed by the loop of the curve

**( b )** Use integration to find the area of $R$

**[ 6 marks ]**

Help for Q1 : http://www.NumberWonder.co.uk/Video/v9045(11b).mp4

**Question 2**

*A-Level Examination Question from June 2018, Q14 (Edexcel)*

A curve $C$ has parametric equations

$$x = 3 + 2\sin t \qquad y = 4 + 2\cos 2t \qquad 0 \leqslant t < 2\pi$$

**( a )** Show that all points on $C$ satisfy

$$y = 6 - (x - 3)^2$$

**[ 2 marks ]**

**( b )** **( i )** Sketch the curve $C$

**( ii )** Explain briefly why $C$ does not include all points of

$$y = 6 - (x - 3)^2 \qquad x \in \mathbb{R}$$

**[ 3 marks ]**

The line with equation $x + y = k$ where $k$ is a constant, intersects $C$ at two distinct points

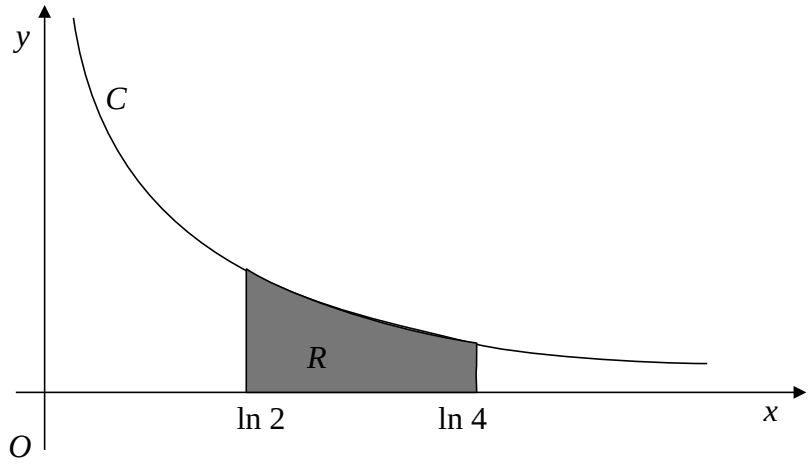
**( c )** State the range of values of $k$ writing your answer in set notation

**[ 5 marks ]**

Help for Q2 : http://www.NumberWonder.co.uk/Video/v9045(11c).mp4

**Question 3**

*A-Level Examination Question from January 2008, Paper C4 (Edexcel)*




The curve $C$ has parametric equations

$$x = ln\,(t + 2) \qquad y = \frac{1}{(t + 1)}$$

The finite region $R$ between the curve $C$ and the $x$-axis, bounded by the lines with equations $x = ln\,2$ and $x = ln\,4$, is shown shaded.

**( a )** Show that the area of $R$ is given by the integral,

$$\int_0^2 \frac{1}{(t + 1)\,(t + 2)}\,dt$$

**[ 4 marks ]**

**( b )** Hence find an exact value for this area.

**[ 6 marks ]**

**( c )** Find a Cartesian equation of the curve $C$ in the form $y = f(x)$

**[ 4 marks ]**

**( d )** State the domain of values for $x$ for this curve.

**[ 1 mark ]**

Help for Q3 : http://www.NumberWonder.co.uk/Video/v9045(11d).mp4