8.1 The Trapezium Rule

Integrating algebraically is desirable but not all curves have equations that can be integrated in this way. For such cases, numerical methods can be used. These are less informative because a solution method that uses numbers rather than algebra gives far less information about the bigger picture in which the solution is embedded. One such numerical approximation method is called the trapezium rule.

8.2 The Formulae Book

The trapezium rule is given in the examination formulae booklet;

$$\int_a^b y\,dx \approx \frac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \dots + y_{n-1})\}, \text{ where } h = \frac{b-a}{n}$$

The h is termed the strip width. The greater the value of n the more accurate the numerical approximation is to the true answer. However, as n is increased, the numerical calculation takes longer.

8.3 Example

The following is a table of values for $y = \sqrt{\sin x}$ where x is in radians.

x	0	$\frac{\pi}{8}$	$\frac{2\pi}{8}$	$\frac{3\pi}{8}$	$\frac{4\pi}{8}$
y	0	0.383	p	0.924	q

(i) Find the value of p and the value of q

[2 marks]

(ii) Use the trapezium rule and all the values of y in the completed table to obtain an estimate of A, where

$$A = \int_0^{\frac{\pi}{2}} \sqrt{\sin x}\,dx$$

[3 marks]

(iii) Hence determine an estimate of the related quantity, B, where

$$B = \int_0^{\frac{\pi}{2}} \left(\sqrt{\sin x} + 3\right) dx$$

[2 marks]

8.4 Exercise

Show sufficient working to make your methods clear.

Marks Available : 40

Question 1

The following is a table of values for $y = \sqrt{1 + \sin x}$ where x is in radians.

x	0	0.5	1	1.5	2
y	1	1.216	p	1.413	q

(i) Find the value of p and the value of q

[2 marks]

(ii) Use the trapezium rule and all the values of y in the completed table to obtain an estimate of A, where,

$$A = \int_0^2 \sqrt{1 + \sin x}\, dx$$

[3 marks]

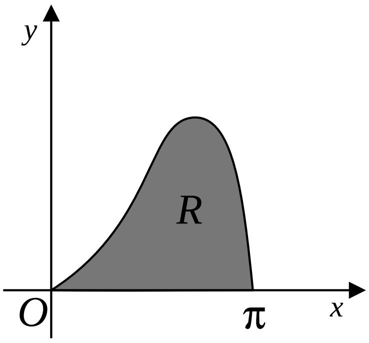
(iii) Hence determine an estimate of the related quantity, B, where,

$$B = \int_0^2 x + \sqrt{1 + \sin x}\ \, dx$$

[2 marks]

Question 2

A-Level Examination Question from January 2008, Paper C4, Q1 (Edexcel)



The curve shown has equation, $y = e^x\sqrt{\sin x}$ $\quad 0 \leqslant x \leqslant \pi$

The finite region R bounded by the curve and the x-axis is shown shaded.

(a) Complete the table with the values of y, correct to 5 decimal places.

x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
y	0			8.87207	

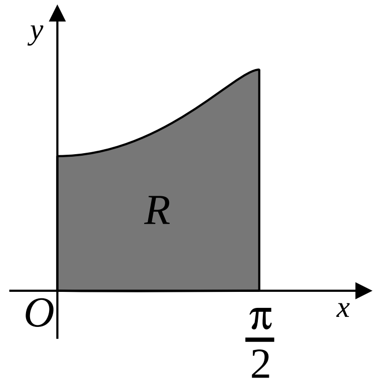
[2 marks]

(b) Use the trapezium rule, with all the values in the completed table, to obtain an estimate for the area of the region R.
Give your answer to 4 decimal places.

[4 marks]

Question 3

A-Level Examination Question from June 2013, Paper C4, Q3 (edit) (Edexcel)



The curve shows the finite region R bounded by the x-axis, the y-axis, the line $x = \dfrac{\pi}{2}$ and the curve with equation, $y = sec\left(\dfrac{1}{2}x\right) \quad 0 \leqslant x \leqslant \dfrac{\pi}{2}$

(a) Complete the table, giving the missing values of y to 6 decimal places.

x	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
y	1	1.035276		1.414214

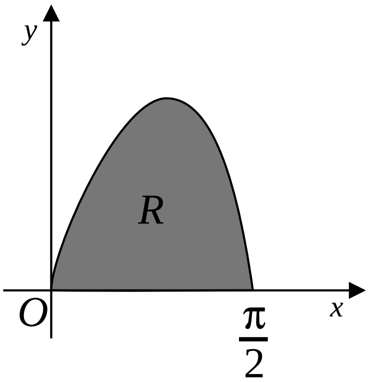
[1 mark]

(b) Using the trapezium rule, with all the values of y from the completed table, find an approximation for the area of R.
Give your answer to 4 decimal places.

[3 marks]

Question 4

A-Level Examination Question from January 2012, Paper C4, Q6 (edit) (Edexcel)



The sketch is of the curve with equation, $y = \dfrac{2\sin 2x}{(1 + \cos x)}$ $\quad 0 \leqslant x \leqslant \dfrac{\pi}{2}$

(a) Complete the table, giving the missing values of y to 5 decimal places.

x	0	$\frac{\pi}{8}$	$\frac{\pi}{4}$	$\frac{3\pi}{8}$	$\frac{\pi}{2}$
y	0		1.17157	1.02280	0

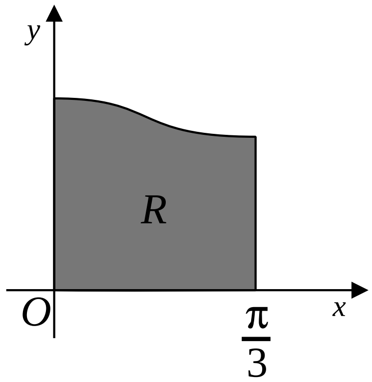
[1 mark]

(b) Use the trapezium rule, with all the values of y from the completed table, to obtain an estimate for the area of R.
Give your answer to 4 decimal places.

[3 marks]

Question 5

A-Level Examination Question from June 2010, Paper C4, Q1 (edit) (Edexcel)



The sketch is of the curve with equation, $y = \sqrt{0.75 + \cos^2 x}$

The finite region R, shown shaded, is bounded by the curve, the y-axis, the x-axis and the line with equation $x = \frac{\pi}{3}$

(a) Complete the table with values of y corresponding to $x = \frac{\pi}{6}$ and $x = \frac{\pi}{4}$

x	0	$\frac{\pi}{12}$	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$
y	1.3229	1.2973			1

[2 marks]

(b) Use the trapezium rule,

(i) with the values of y at $x = 0$, $x = \frac{\pi}{6}$ and $x = \frac{\pi}{3}$ to find an estimate of the area of R. Give your answer to 3 decimal places.

[3 marks]

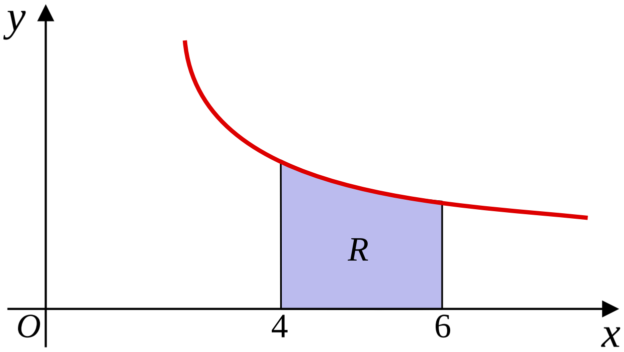
(ii) with the values of y at $x = 0$, $x = \frac{\pi}{12}$, $x = \frac{\pi}{6}$, $x = \frac{\pi}{4}$ and $x = \frac{\pi}{3}$ to find a further estimate of the area of R.

Give your answer to 3 decimal places.

[3 marks]

Question 6

A-Level Examination Question from January 2019, Paper C34, Q7 (Edexcel)



The diagram shows a sketch of part of the curve with equation,

$$y = \frac{x + 7}{\sqrt{2x - 3}}, \qquad x > \frac{3}{2}$$

The region R, shown shaded, is bounded by the curve, the line with equation $x = 4$, the x-axis and the line with equation $x = 6$

(a) Use the trapezium rule with 4 strips of equal width to find an estimate for the area of R, giving your answer to 2 decimal places.

[4 marks]

(b) Using the substitution $u = 2x - 3$, or otherwise, use calculus to find the exact area of R, giving your answer in the form $a + b\sqrt{5}$, where a and b are constants to be found.

[7 marks]