Lesson 6

Further A-Level Pure Mathematics, Core 2

Differential Equations II

6.1 Complementary Function + Particular Integral

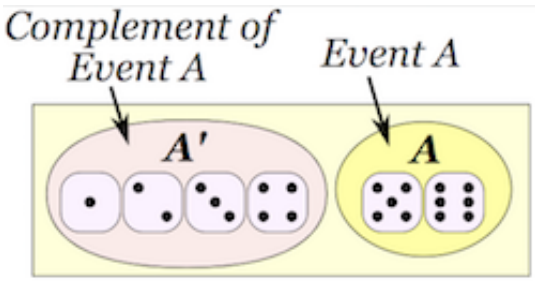
Via consideration of the discriminant of the auxiliary equation, we have developed techniques to solve second-order, linear, homogeneous, differential equations with constant coefficients. That is, equations of the form,

$$a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = 0$$

It is the zero on the right hand side that earned it the descriptor "homogeneous". With not much extra work, we can extend our repertoire to handle the situation where the zero is replaced with a function of x. In other words, "nonhomogeneous" differential equations of the form,

$$a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = f(x)$$

To solve this type of nonhomogeneous equation, first find the general solution of the corresponding homogeneous differential equation. This general solution is called the complementary function, CF.



The hunt is then on to find a particular integral, PI, which is a function that satisfied the nonhomogeneous differential equation. Here we take the advice of mathematicians that have gone before us. Their collective wisdom is contained in the following table. Depending on the nature of $f(x)$ they suggest what form the particular integral will take.

Form of $f(x)$	Form of PI
u	U
$ux + v$	$Ux + V$
$u x^r + \ldots + vx + w$	$U x^r + \ldots + Vx + W$
$u \cos kx$	$U \cos kx + V \sin kx$
$v \sin kx$	$U \cos kx + V \sin kx$
$u \cos kx + v \sin kx$	$U \cos kx + V \sin kx$
$u e^{kx}$	$U e^{kx}$
$u e^{-kx}$	$U e^{-kx}$

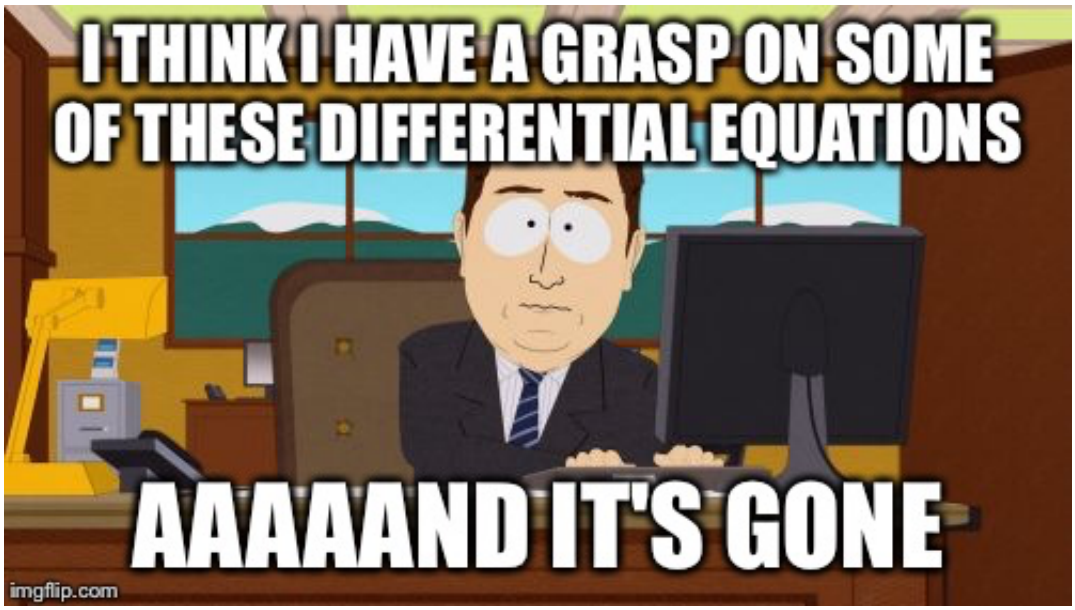
The general solution to the nonhomogeneous equation is then CF + PI

6.2 Example

Find the general solution to the differential equation,

$$\frac{d^2y}{dx^2} - 8\frac{dy}{dx} + 12y = 36x$$

[6 marks]



6.3 Exercise

Any solution based entirely on graphical or numerical methods is not acceptable

Marks Available : 40

Question 1

Find the general solution to the differential equation,

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 3y = 6$$

[6 marks]

Question 2

Find the general solution to the differential equation,

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 5e^{3x}$$

[6 marks]

Question 3

Find the general solution to the differential equation,

$$\frac{d^2y}{dx^2} - y = 2x^2 - x - 3$$

[6 marks]

Question 4

Find the general solution to the differential equation,

$$\frac{d^2y}{dx^2} + 4y = \sin x$$

[6 marks]

Question 5

Find the general solution to the differential equation,

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 25\cos 2x$$

[7 marks]

Question 6

Further A-Level Examination Question from June 2012, Paper FP2, Q4 (Edexcel)

Find the general solution to the differential equation,

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = 2\cos t - \sin t$$

[9 marks]