

## Lesson 9

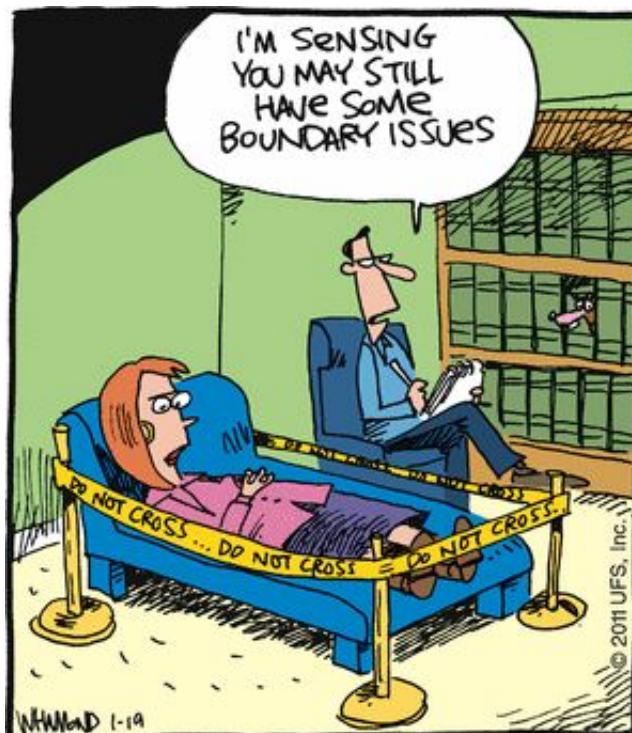
### Further A-Level Pure Mathematics, Core 2 Differential Equations II

#### 9.1 Boundary Conditions

The general solution to a second order differential equation typically involves some arbitrary constants that we have tended to label  $A$  and  $B$ . A question may provide some extra information, some boundary conditions, that allow the particular solution to be found, that is, to work out numerical values for  $A$  and  $B$ . Usually, two pieces of extra information will be given. Each piece may yield the value one of the arbitrary constants or you may need to set up and solve two simultaneous equations to get the values.

#### 9.2 Example

Given that  $y'(0) = 1$  and  $y(0) = 0$ , find the particular solution to the differential equation  $y'' + 5y' + 4y = 4$



[ 8 marks ]

### 9.3 Exercise

*Any solution based entirely on graphical  
or numerical methods is not acceptable*

Marks Available : 50

#### Question 1

Given that  $y'(0) = 18$  and  $y(0) = 36$ , find the particular solution to the differential equation  $y'' + 7y' - 8y = 72$

[ 8 marks ]

**Question 2**

Given that  $y'(0) = 0$  and  $y(0) = 4$ , find the particular solution to the differential equation  $y'' + 12y' + 37y = 10e^{-4x}$

[ 9 marks ]

**Question 3**

Find  $y$  in terms of  $k$  and  $x$ , given that  $\frac{d^2y}{dx^2} + k^2 y = 0$  where  $k$  is a constant,

and  $y = 1$  and  $\frac{dy}{dx} = 1$  at  $x = 0$

[ 8 marks ]

**Question 4**

*Further A-Level Examination Question from June 2014, FP2, Q5 (Edexcel)*

( a ) Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} + 2 \frac{dy}{dx} + 10y = 27 e^{-x}$$

[ 6 marks ]

( b ) Find the particular solution that satisfies  $y = 0$  and  $\frac{dy}{dx} = 0$  when  $x = 0$

[ 6 marks ]

**Question 5**

*Further A-Level Examination Question from June 2017, FP2, Q5 (Edexcel)*

( a ) Find the general solution of the differential equation,

$$\frac{d^2y}{dx^2} - 2 \frac{dy}{dx} = 26 \sin 3x$$

[ 8 marks ]

(b) Find the particular solution of this differential equation for which

$$y = 0 \text{ and } \frac{dy}{dx} = 0 \text{ when } x = 0$$

[ 5 marks ]

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