**GCSE** Mathematics

# I TERATION I TERATION



THE COLLATZ CONJECTURE STATES THAT IF YOU PICK A NUMBER, AND IF IT'S EVEN DIVIDE IT BY TWO AND IF IT'S ODD MULTIPLY IT BY THREE AND ADD ONE, AND YOU REPEAT THIS PROCEDURE LONG ENOUGH, EVENTUALLY YOUR FRIENDS WILL STOP CALLING TO SEE IF YOU WANT TO HANG OUT.

# ITERATION

### Lesson 1

## GCSE Mathematics Iteration

## **1.1 What is a Iteration ?**

An iterative process is one in which you repeatedly carry out the same set of instructions. The idea of iteration has been around for over two hundred years but it is only since the invention of the desktop computer in the 1980s that the subject has become mainstream mathematics. A flowchart provides a convenient way of describing an iteration.

## 1.2 Example

Consider the following flowchart,



This flowchart generates a sequence of numbers.

We can give the sequence a name; sequence U.

The first term in sequence U is denoted  $U_1$ 

The second term in sequence U is denoted  $U_2$ 

And so on...

Complete this table to show the first eight terms in sequence U

${U}_1$	$U_2$	$U_3$	${U}_4$	$U_5$	$U_6$	$U_7$	${U}_8$
34							

[6 marks]

## 1.3 Exercise

## **Non-Calculator**

Marks Available : 40

## **Question 1**

Consider the following flowchart,



This flowchart generates a sequence of numbers.

We can give the sequence a name; sequence V

The first term in sequence V is denoted  $V_1$ 

The second term in sequence V is denoted  $V_2$ 

And so on...

Complete this table to show the first seven terms in sequence V

$V_1$	$V_2$	$V_3$	$V_4$	$V_5$	$V_6$	$V_7$
366						

[6 marks]

## **Question 2**

This question will tell you how to get terms in sequence Q. Start with the number 1, that is  $Q_1 = 1$ To get a next term, double the previous term. What is  $Q_6$ ?

[ 2 marks ]

Consider the following flowchart,



This flowchart generates the sequence of numbers D. Complete this table to show the first seven terms in sequence D

$D_1$	$D_2$	$D_3$	$D_4$	$D_5$	$D_6$	$D_7$
189						

[ 6 marks ]

## **Question 4**

This question will tell you how to get terms in sequence A. Start with the number one million, that is  $A_1 = 1000000$ To get a next term, divide the previous term by 10.

(i)	What is the	value of $A_2$ ?
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- (ii) What is the value of  $A_3$ ?
- (**iii**) What is the value of  $A_7$ ?

(iv) What is the value of  $A_9$ ?

[1 mark]

[1 mark]

[1 mark]

Consider the following flowchart,



This flowchart generates the sequence of numbers *P* Complete this table to show the first ten terms in sequence *P* 

$P_1$	$P_2$	<i>P</i> <sub>3</sub>	$P_4$	$P_5$	$P_6$	$P_7$	$P_8$	$P_9$	<i>P</i> <sub>10</sub>
			7						

**Question 6** 



In my garden on Monday at noon there are 100 snails.

At noon each day, there are 20% more snails than at noon the day before.

How many snails are in my garden at noon on Wednesday that week ?

[ 3 marks ]

[6 marks]



This flowchart which behaves in a different manner to the others in this exercise. Call the sequence of numbers generated by this flowchart ZComplete this table to show the first ten terms in sequence Z

$Z_1$	$Z_2$	$Z_3$	$Z_4$	$Z_5$	$Z_6$	$Z_7$	$Z_8$	$Z_9$	$Z_{10}$

[6 marks]

## **Question 8**

Explain how to modify this "scratch" programming loop to draw a hexagon.



[ 2 marks ]

See if you can you handle this question without using a calculator. (It's all to do with powers of 2)



This flowchart generates the sequence of numbers CComplete this table to show the first five terms in sequence C

$P_1$	$P_2$	$P_3$	$P_4$	$P_5$
$\frac{1}{16}$				

[ 4 marks ]

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Teachers may obtain detailed worked solutions to the exercises by email from mhh@shrewsbury.org.uk