3.1 Coding The Flowchart

Although a flowchart is an easily understood way of describing an iterative process it takes up a lot of space on the page and is time consuming to draw. Mathematicians have devised a much more clever way of describing an iteration. Here's how they translate two key phrases about a sequence U into mathematics,

"the term of focus" becomes U_n

"the next term (one step on from the term of focus)" becomes U_{n+1}

Complete the following table;

n	U_n	U_{n+1}
1	U_1	$U_{1+1} = U_2$
2		
3		
...		
8		
...		
43		

[3 marks]

3.2 Example

A number sequence, U, has the following iterative description,

$$U_1 = 0 \qquad U_{n+1} = 3U_n + 1$$

Note the difference in meaning between $n + 1$ in a small font size with that in larger. Complete the table to show the first eight terms of the sequence.

U_1	U_2	U_3	U_4	U_5	U_6	U_7

[4 marks]

3.3 Exercise

Non-Calculator

Marks Available : 50

Question 1

A number sequence, *A*, has the following iterative description,

$$A_1 = 2 \qquad A_{n+1} = 2A_n - 1$$

Complete the table to show the first seven terms of the sequence.

A_1	A_2	A_3	A_4	A_5	A_6	A_7

[4 marks]

Question 2

A number sequence, *H*, has the following iterative description,

$$H_1 = 11 \qquad H_{n+1} = 2A_n - 10$$

Complete the table to show the first eight terms of the sequence.

H_1	H_2	H_3	H_4	H_5	H_6	H_7	H_8

[4 marks]

Question 3

Professor RE Peat believes that the following iteration will always generate a prime number.

$$P_1 = 5 \qquad P_{n+1} = 2P_n - 3$$

(i) Complete the table to show the first six terms of the sequence.

P_1	P_2	P_3	P_4	P_5	P_6

[4 marks]

(ii) Considering just the six terms from part (i), might the professor be correct ? Give a reason for your answer.

[2 marks]

Question 4

A number sequence, B, has the following iterative description,

$$B_1 = 10 \qquad B_{n+1} = 3B_n - 10$$

Complete the table to show the first seven terms of the sequence.

B_1	B_2	B_3	B_4	B_5	B_6	B_7

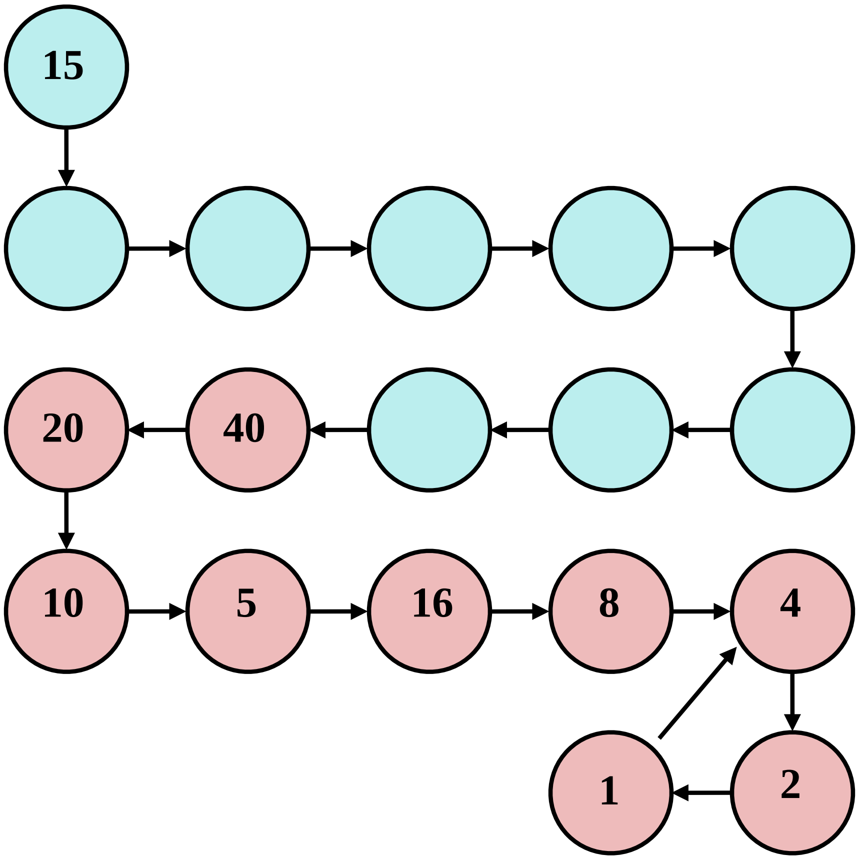
[4 marks]

Question 5

Last lesson we looked at an unsolved mathematics problem; the Collatz conjecture.
It had a flowchart that did a different calculation for the next term in the sequence depending upon if the "number in mind" was even or odd.
Here is how that flowchart is described mathematically,

$$C_{n+1} = \begin{cases} \dfrac{C_n}{2} & \text{if } C_n \text{ is even} \\ 3C_n + 1 & \text{if } C_n \text{ is odd} \end{cases}$$

Starting with 15, work out the 8 missing terms before this new branch of tree joins onto the main trunk (worked out last lesson).



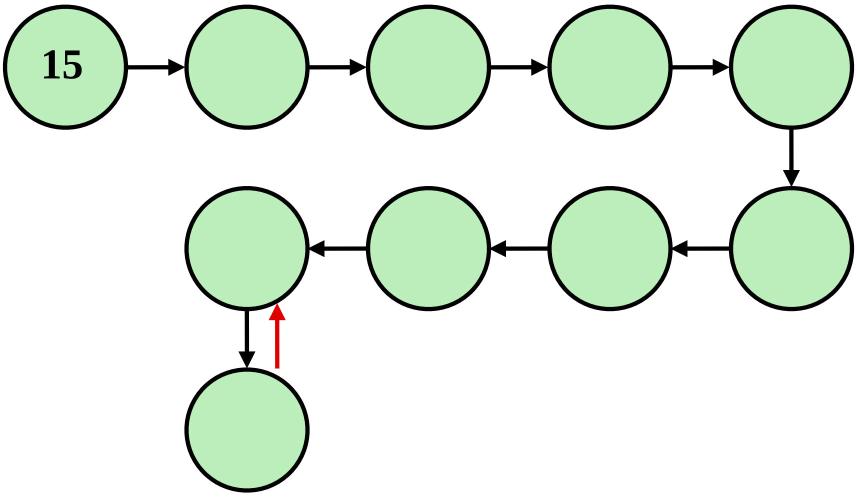
[6 marks]

Question 6

It's natural to wonder if the Collatz conjecture holds for other similar iterations. Consider this very similar rule where the "add 1" is changed to "subtract 1".

$$L_{n+1} = \begin{cases} \dfrac{L_n}{2} & \text{if } L_n \text{ is even} \\ 3\,L_n - \mathbf{1} & \text{if } L_n \text{ is odd} \end{cases}$$

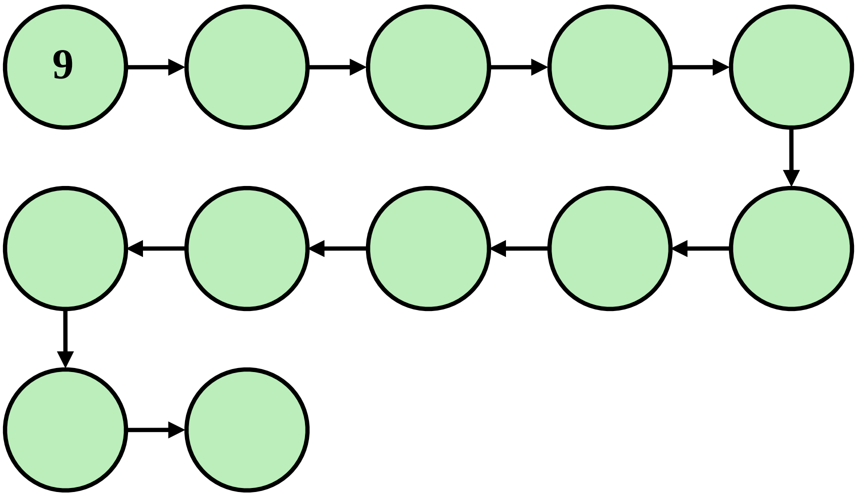
(i) On the following diagram write out the numbers generated.if $L_1 = 15$



This suggests that the new rule may be behaving much like the old.

[6 marks]

(ii) On the following diagram write out the numbers generated.if $L_1 = 9$



[6 marks]

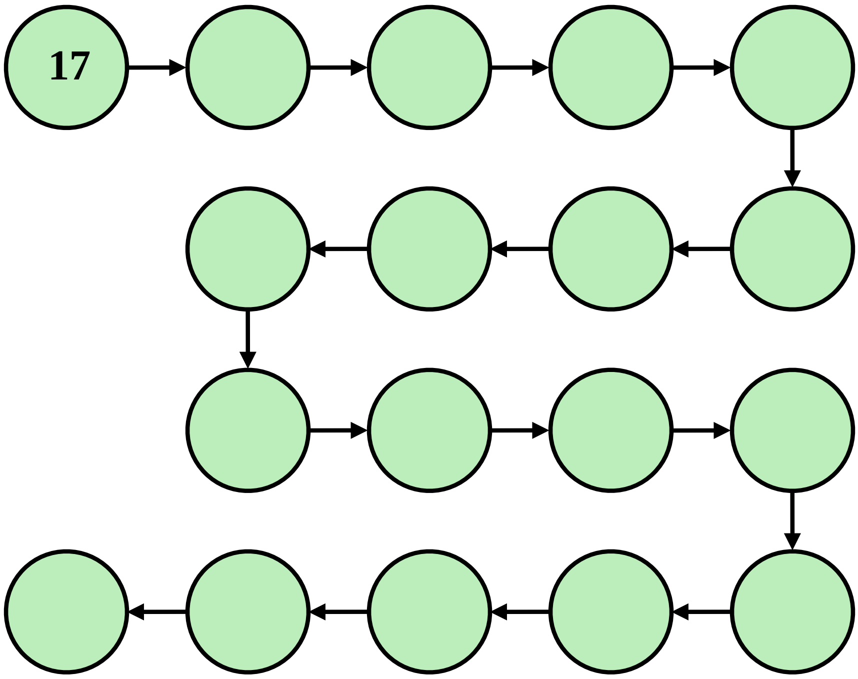
(iii) Add an arrow to the last circle of your part (ii) answer to show how it connects back to a previous circle in the sequence.

[1 mark]

(iv) Explain how your part (ii) and (iii) answers show that the Collatz conjecture is FALSE for the adjusted rule.

[2 marks]

(v) On the following diagram write out the numbers generated.if $L_1 = 17$



[10 marks]

(vi) Add an arrow to the last circle of your part (v) answer to show how it connects back to a previous circle in the sequence.

[1 mark]