

5.1 The Many Faces of One



In mathematics, all numbers are important but some are more special than others. In today's lesson extensive use will be made of the fact that when anything is multiplied by one it does nothing.

You may be wondering why, if multiplying by one “does nothing”, it's of any use at all. The usefulness of one comes from the fact that it has many faces.

5.2 Example

Determine the first six terms of the sequence with term-to-term description,

$$U_1 = \frac{1}{3}, \quad U_{n+1} = \frac{2 - U_n}{2}$$

U_1	U_2	U_3	U_4	U_5	U_6

[7 marks]

5.3 Exercise

Non-Calculator
Marks Available : 50

Question 1

Simplify,

(i) $14 \times \frac{5}{7}$

(ii) $5\left(3 + \frac{1}{5}\right)$

(iii) $\left(\frac{5}{7} + 3\right) \times 7$

(iv) $\left(7 + \frac{3}{4}\right) \times 4$

[4 marks]

Question 2

Each of the following calculations has an answer that is a rational number.

That is, a number in the form $\frac{p}{q}$ for integer p and q with $q \neq 0$

For each, determine what that rational number is.

(i) $\frac{\left(1 + \frac{1}{2}\right)}{13} \times \frac{2}{2}$

[2 marks]

(ii) $\frac{\left(3 + \frac{1}{4}\right)}{7} \times \frac{4}{4}$

[2 marks]

(iii) $\frac{\left(\frac{5}{7} + 2\right)}{6} \times \frac{7}{7}$

[2 marks]

(iv) $\frac{\left(11 + \frac{5}{9}\right)}{7}$

[2 marks]

(v) $\frac{\left(4 + \frac{3}{8}\right)}{8}$

[2 marks]

(vi) $\frac{\left(\frac{8}{3} + 7\right)}{4}$

[2 marks]

Question 3

- (i) The following sum has an answer that is a rational number.

That is, a number in the form $\frac{p}{q}$ for integer p and q with $q \neq 0$

Determine what that rational number is.

$$\frac{\left(3 - \frac{1}{2}\right)}{2}$$

[2 marks]

- (ii) Consider the iteration, $A_1 = \frac{1}{2}$, $A_{n+1} = \frac{3 - A_n}{2}$

Use the space below to work out the first six terms of this iterative sequence and put your answers in the table towards the bottom of the page.

A_1	A_2	A_3	A_4	A_5	A_6

[7 marks]

- (iii) From looking at your table of results, and spotting a pattern, write down what the next three terms, A_7 , A_8 and A_9 are likely to be.

[2 marks]

Question 4

(i) The following sum has an answer that is a rational number.

That is, a number in the form $\frac{p}{q}$ for integer p and q with $q \neq 0$

Determine what that rational number is.

$$\frac{\left(1 - \frac{3}{10}\right)}{2}$$

[2 marks]

(ii) Consider the iteration, $B_1 = \frac{3}{10}$, $B_{n+1} = \frac{1 - B_n}{2}$

Use the space below to work out the first six terms of this iterative sequence and put your answers in the table towards the bottom of the page.

B_1	B_2	B_3	B_4	B_5	B_6

[7 marks]

(iii) From looking at your table of results, and spotting a pattern, write down what the next three terms, B_7 , B_8 and B_9 are likely to be.

[2 marks]

Question 5

- (i) The following sum has an answer that is a rational number.

That is, a number in the form $\frac{p}{q}$ for integer p and q with $q \neq 0$

Determine what that rational number is.

$$\frac{\left(1 - \frac{2}{3}\right)}{3}$$

[2 marks]

- (ii) Consider the iteration, $Z_1 = \frac{2}{3}$, $Z_{n+1} = \frac{1 - Z_n}{3}$

Use the space below to work out the first six terms of this iterative sequence and put your answers in the table towards the bottom of the page.

Z_1	Z_2	Z_3	Z_4	Z_5	Z_6

[7 marks]

- (iii) From looking at your table of results, and spotting a pattern, write down what the next three terms, Z_7 , Z_8 and Z_9 are likely to be.

[3 marks]

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Teachers may obtain detailed worked solutions to the exercises by email from mhh@shrewsbury.org.uk