SHREWSBURY SCHOOL.

MATHEMATICS PRIZE.
1946.

1. Two cars are 1,210 ft. apart on a straight road moving in the same direction. A quarter of a mile ahead is a hill on which the speed of each will be reduced by a half. The faster car overtakes the other after travelling 2,200 ft. If the speed of the faster car is 45 miles per hour on the level, what is the speed of the slower ?

2. The following discounts are made on an insurance premium. (i) $p\%$ no claim bonus, (ii) $q\%$ agent's discount, (iii) $r\%$ special war-time reduction. Show that if they are applied in succession the final sum to be paid is independent of the order of their application.
   If the total effect of these three discounts is the same as that of a single discount of $x\%$ find x in terms of p, q and r.

3. ABC is a triangle. On AB is constructed an isosceles triangle APB, having angle APB a right angle. Q is a point on AB such that AQ = AP. Prove that a line through Q parallel to BC bisects the area of the triangle.

4. A cylinder of radius 4 ins. contains water to a depth of $7\frac{7}{24}$ ins. A right circular cone whose height is 15 ins. and whose base has radius 3 ins. is placed in the cylinder with vertex upwards. How high does the water rise ?

5. (i) Simplify

$$\frac{\left(\frac{x^2}{y^2}+\frac{y^2}{x^2}+1\right)}{\left(\frac{x}{y}+\frac{y}{x}+1\right)} \times \frac{\left(\frac{x^2}{y}+\frac{y^2}{x}+\frac{1}{xy}-3\right)}{(x-y)^2+(y-1)^2+(x-1)^2} \div (x^3+y^3+x^2+y^2-xy)$$

   (ii) If $$\frac{a}{x+y-z}=\frac{b}{y+z-x}=\frac{c}{z+x-y}$$

   Prove that $$\frac{2a+b+c}{b+c}=\frac{x+y}{z}$$

6. A lamp shade has the following dimensions : radius of top 1 ft., radius of bottom 1 ft. 4 ins., slant height 4 ft. A fly crawls from a point on the top edge to the nearest point on the bottom edge but makes one complete circuit of the shade on the way down. Find the length of its shortest path.

7. ABCD is a paralellogram. The diagonal BD is divided at P, Q, R, S into five equal parts. Join AP and produce to cut BD at W. Join WQ and produce to cut AD at X. Join XR and produce to cut BC at Y. Join YS and produce to cut AD at Z. If AD is 64 ins. long, find the length of ZD.

8. Given that $x^2+x+1=a$
   Prove that $x^8+x^4+1=a^4-4a^3x+2a^2x+2a^3-6a^2+4a$.

9. ABCD is a paralellogram in which AD is 10 ins. and the angle A 130°. The diagonals meet at O and the angle AOD is 70°.