

Lesson 3

GCSE Mathematics Ratio and Similarity

Non Calculator

3.1 Start Up

Use the table of *Squares and Cubes from 1 to 99* to determine;

(i)

$$\left(\frac{21}{26}\right)^2 = \frac{441}{\quad}$$

(ii)

$$\left(\frac{49}{34}\right)^2 = \frac{\quad}{1156}$$

(iii)

$$\left(\frac{32}{91}\right)^2 = \frac{\quad}{\quad}$$

(iv)

$$\left(\frac{17}{31}\right)^3 = \frac{4913}{\quad}$$

(v)

$$\left(\frac{7}{29}\right)^3 = \frac{\quad}{24389}$$

(vi)

$$\left(\frac{63}{50}\right)^3 = \frac{\quad}{\quad}$$

(vii)

$$\left(\frac{324}{625}\right)^{0.5} = \frac{18}{\quad}$$

(viii)

$$\left(\frac{196}{4356}\right)^{0.5} = \frac{\quad}{66}$$

(ix)

$$\left(\frac{6241}{9801}\right)^{0.5} = \frac{\quad}{\quad}$$

(x)

$$\left(\frac{81}{25}\right)^{\frac{1}{2}} = \frac{\quad}{\quad}$$

(xi)

$$\left(\frac{1}{100}\right)^{\frac{1}{2}} = \frac{\quad}{\quad}$$

(xii)

$$\left(\frac{2116}{2601}\right)^{\frac{1}{2}} = \frac{\quad}{\quad}$$

(xiii)

$$\left(\frac{8}{27}\right)^{\frac{1}{3}} = \frac{\quad}{\quad}$$

(xiv)

$$\left(\frac{4096}{24389}\right)^{\frac{1}{3}} = \frac{\quad}{\quad}$$

(xv)

$$\left(\frac{857375}{884736}\right)^{\frac{1}{3}} = \frac{\quad}{\quad}$$

[15 marks]

3.2 Volume Scale Factor (vsf)

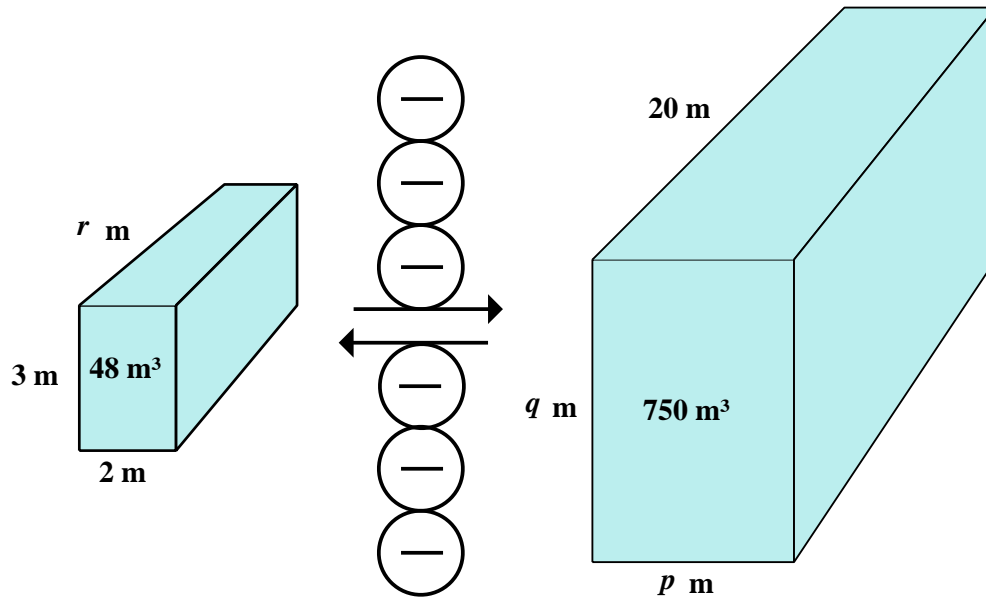
For any two similar solids:

$$\text{volume scale factor} = (\text{length scale factor})^3$$

which can also be expressed as:

$$\text{length scale factor} = \sqrt[3]{\text{volume scale factor}}$$

3.3 Example



The above two similar cuboids are shown with the same orientation.

- (i) Find the lengths marked p , q and r .

[3 marks]

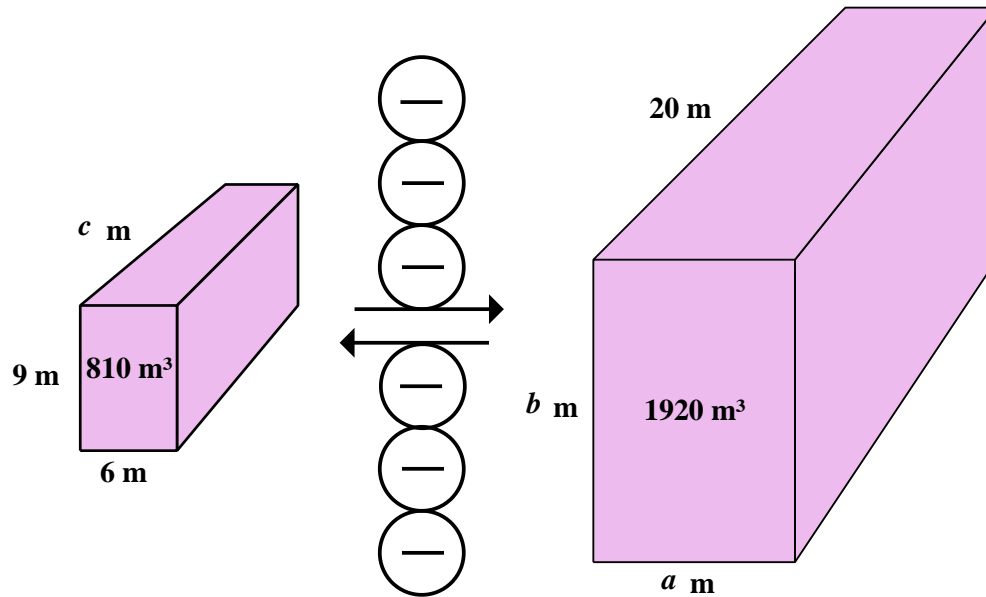
- (ii) How many times more surface area has the larger cuboid than the smaller ?

[1 mark]

3.4 Exercise

Marks Available : 56

Question 1



The above two similar cuboids are shown with the same orientation.

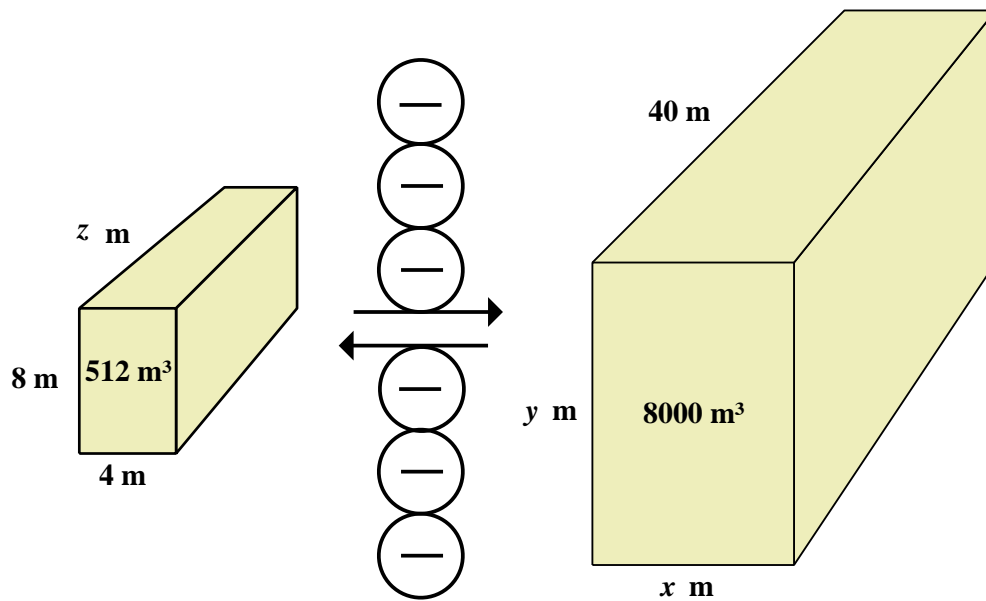
- (i) Find the lengths marked a , b and c .

[3 marks]

- (ii) How many times more surface area has the larger cuboid than the smaller ?

[2 marks]

Question 2



The above two similar cuboids are shown with the same orientation.

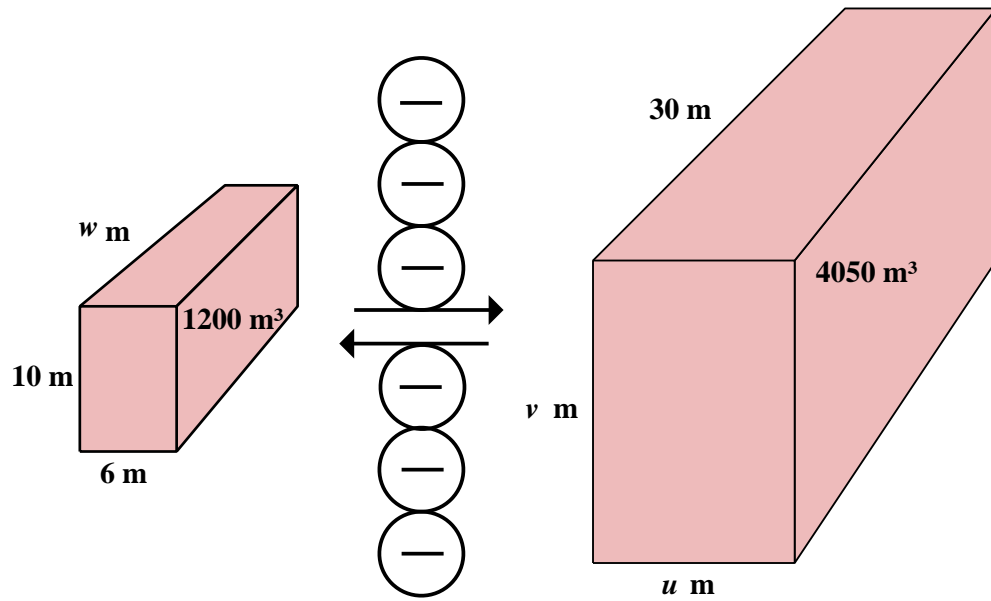
- (i) Find the lengths marked x , y and z .

[3 marks]

- (ii) How many times more surface area has the larger cuboid than the smaller ?

[2 marks]

Question 3



The above two similar cuboids are shown with the same orientation.

- (i) Find the lengths marked u , v and w .

[3 marks]

- (ii) How many times more surface area has the larger cuboid than the smaller ?

[2 marks]

Question 4

- (i) Are all squares similar ? YES / NO

In other words, *Are all squares the same shape ?*

[1 mark]

- (ii) A smaller square has an area of 27cm^2 .

A larger square has a side $\frac{5}{3}$ times greater than the small square.

What is the area of the larger square ?

$$\text{Hint : } lsf = \frac{5}{3} \quad asf = (lsf)^2 \quad A_{BIG} = A_{SMALL} \times asf$$

[3 marks]

Question 5

Cuboid *G* measures 14cm by 21cm by 28cm.

Cuboid *H* measures 4cm by 6cm by 8cm.

- (i) Complete the following to show that the two cuboids similar.

$$\frac{14}{6} = \frac{\quad}{\quad} = \frac{\quad}{\quad} \quad \text{All cancel down to } \frac{\quad}{\quad}$$

[2 marks]

- (ii) What is the **length scale factor**, greater than 1, of the similarity ?

$$lsf = \frac{\quad}{\quad}$$

[1 mark]

- (iii) What is the **area scale factor** of, greater than 1, of the similarity ?

$$asf = \frac{\quad}{\quad}$$

[1 mark]

- (iv) What is the **volume scale factor**, greater than 1, of the similarity ?

$$vsf = \frac{\quad}{\quad}$$

[1 mark]

- (v) Calculate;

(a) V_{BIG}

[1 mark]

(b) V_{SMALL}

[1 mark]

(c) $V_{SMALL} \times vsf$

[1 mark]

(d) Comment

[1 mark]

Question 6

Are all rectangles similar ?

YES / NO

In other words, *Are all rectangles the same shape ?*

[1 mark]

Question 7

(i) Are all spheres similar ?

YES / NO

[1 mark]

(ii) A smaller sphere has a volume of 32cm^3 .

A larger sphere has a radius $\frac{5}{2}$ times greater than the small sphere.

What is the volume of the larger sphere ?

$$\text{Hint : } l_{sf} = \frac{5}{2} \quad v_{sf} = (l_{sf})^3 \quad V_{BIG} = V_{SMALL} \times v_{sf}$$

[3 marks]

Question 8

A larger cuboid has lengths that are $\frac{4}{3}$ times longer than a similar smaller cuboid.

The smaller cuboid measures 60cm by 66cm by 42cm.

What are the measurements of the larger cuboid ?

_____ by _____ by _____

[2 marks]

Question 9

A larger cuboid has lengths that are $\frac{7}{4}$ times longer than a similar smaller cuboid.

The smaller cuboid measures 8 cm by 40 cm by 44 cm.

What are the measurements of the larger cuboid ?

_____ by _____ by _____

[2 marks]

Question 10

(i) Are all triangles similar ?

YES / NO

[1 mark]

(ii) Are all right angled triangles similar ?

YES / NO

[1 mark]

(iii) Are all equilateral triangles similar ?

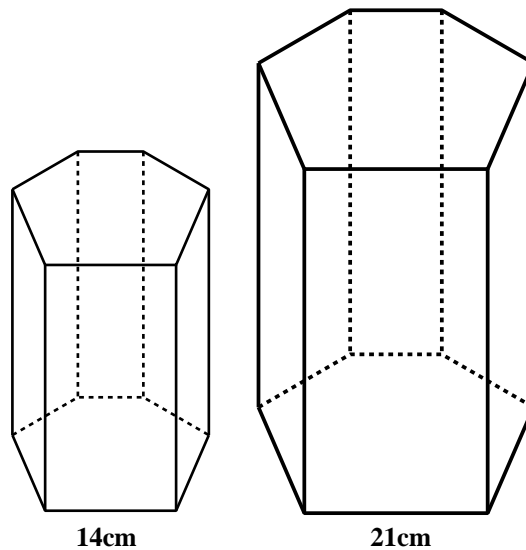
YES / NO

[1 mark]

Question 11

Two similar hexagonal prisms are shown below.

An edge of 14cm on the smaller corresponds to an edge of length 21cm on the larger.



- (i) What is the *length scale factor*, greater than 1, of the similarity ?

$$lsf = \text{———}$$

[1 mark]

- (ii) What is the *area scale factor*, greater than 1, of the similarity ?

$$asf = \text{———}$$

[1 mark]

- (iii) What is the *volume scale factor*, greater than 1, of the similarity ?

$$vsf = \text{———}$$

[1 mark]

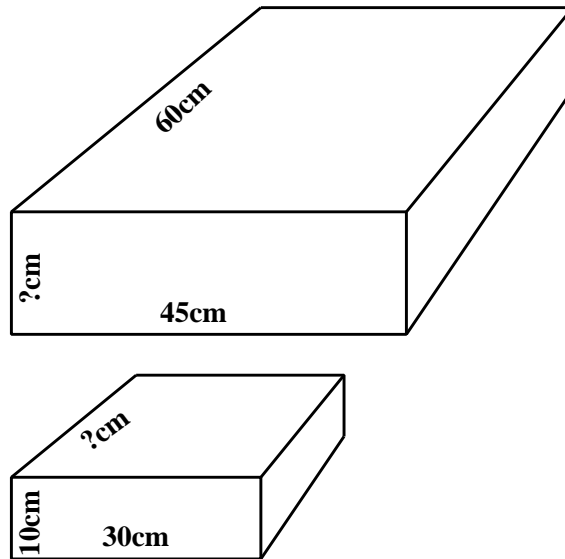
- (iv) The volume of the smaller hexagonal prism is 40 cm^3 .

Calculate the volume of the larger hexagonal prism.

[2 marks]

Question 12

The two cuboids shown below are similar.



- (i) What is the *length scale factor*, greater than 1, of the similarity ?

$lsf = \text{——}$

[1 mark]

- (ii) Find the height of the upper cuboid, marked with a question mark.

[1 mark]

- (iii) Find the missing length on the lower cuboid, marked with a question mark.

[1 mark]

Question 13

Each face of a smaller cube has an area of 36 cm^2

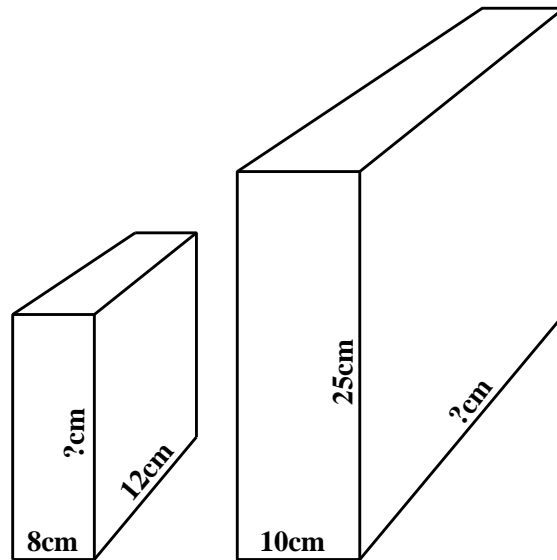
A larger cube has edges that are 10 times longer.

What is the volume of the larger cube ?

[3 marks]

Question 14

The two cuboids shown below are similar.



- (i) What is the *length scale factor*, greater than 1, of the similarity ?

$lsf = \text{———}$

[1 mark]

- (ii) Find the unknown length of the right cuboid, marked with a question mark.

[1 mark]

- (iii) Find the unknown height of the left cuboid, marked with a question mark.

[1 mark]

Question 15

A 250 ml can of coke is similar to a 500 ml can of coke.

Explain why radius of the larger can is NOT double that of the smaller.

[3 marks]