Lesson 4

Number Sequences : Year 9

Non-Calculator

4.1 Arithmetic Progressions

In this Lesson you will discover what an Arithmetic Progression is, and investigate these special number sequences.

4.2 Exercise

Question 1
For each sequence use the *position-to-term* formula to generate the first four terms.

Name	1st term	2nd term	3rd term	4th term	*Position-to-term* formula
B					$B_n = 0.5\,n + 1.5$
D					$D_n = 8\,n - 7$
E					$E_n = 15\,n$
F					$F_n = 0.1\,n - 1$
I					$I_n = 14\,n + 1$
J					$J_n = 2.5\,n - 0.5$

FYI : Read this...
When jumping from one term to the next, if a sequence increases or decreases by the same amount each time, it is said to be in *Arithmetic Progression.*

All of the sequences in the above table are in Arithmetic Progression.

They all have the same sort of formula with “some *n* plus or minus a number”.

Question 2

The Algebraic Form of an Arithmetic Progression.

When jumping from one term to the next in an Arithmetic progression the increase or decrease at each jump is called **the common difference**, d.
The first term is called **the initial term**, a.

For each of the following Arithmetic progressions write down;

(i) the initial term a,

(ii) the common difference, d.

(a) 4, 9, 14, 19, ...

(i) $a =$ **(ii)** $d =$

(b) - 2, 5, 12, 19, ...

(i) $a =$ **(ii)** $d =$

(c) 11, 8, 5, 2, ...,

(i) $a =$ **(ii)** $d =$

(d) $W_n = 11n + 2$

(i) $a =$ **(ii)** $d =$

(e) $X_n = 8n - 5$

(i) $a =$ **(ii)** $d =$

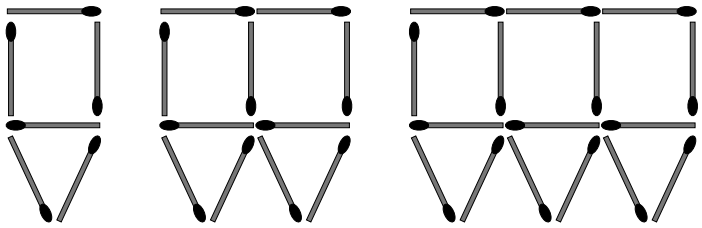
(f) $Y_n = 13 - 3n$

(i) $a =$ **(ii)** $d =$

(g) $Z_n = -5n + 2$

(i) $a =$ **(ii)** $d =$

Question 3



1st **2nd** **3rd**

Matches are used to make the sequence of diagrams shown above.

(a) Complete the following table to show the matches used in each of the three diagrams shown, plus, were the pattern to be continued, the next three.

Position	1 st	2 nd	3 rd	4 th	5 th	6 th
Term						

(b) How many matches would be needed to make the 10 th diagram ?

(c) Is the sequence in Arithmetic progression ?

(d) Try to write down a formula for the n^{th} term of the sequence.

Question 4

A sequence has *initial term* 8 and *common difference* 5.

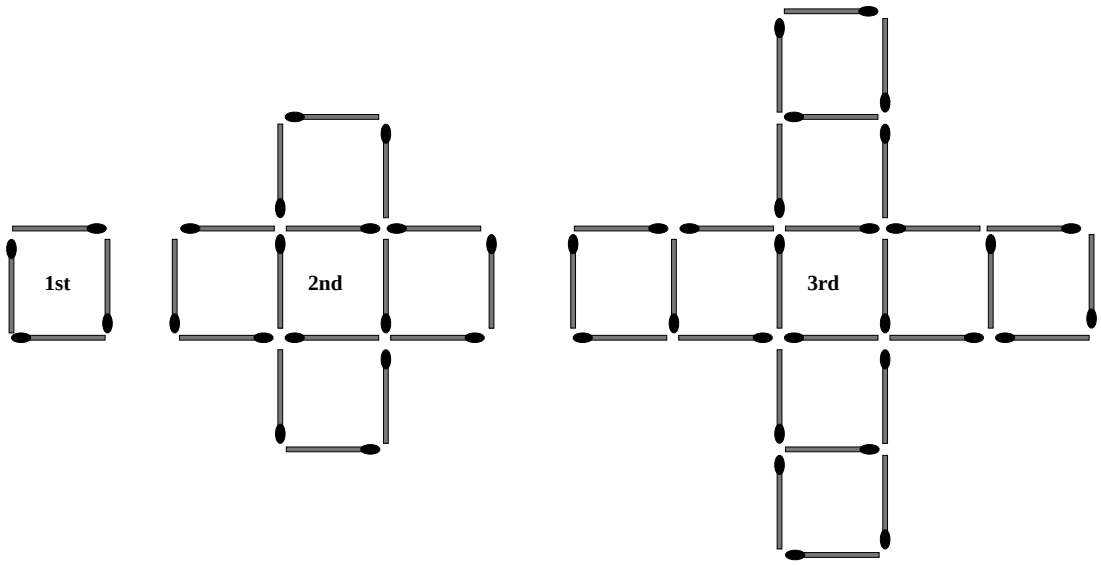
(a) Write out the start of this number sequence in the following table;

Position	1 st	2 nd	3 rd	4 th	5 th	6 th
Term						

(b) Will 1368 be in this number sequence ?

(c) Explain why 1257 can not be a number in this sequence.

Question 5



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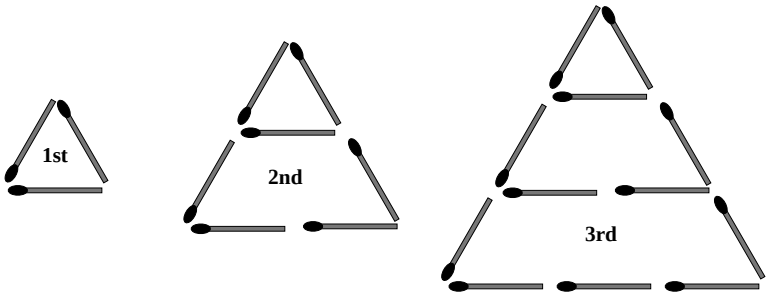
Question 6



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(b) How many matches would be needed to make the 10 th diagram ?

(c) Is the sequence in Arithmetic progression ?

(d) The formula for the sequence is

$$A_n = \frac{1}{2}(n)(n+5)$$

(i) Use this formula to check your answer to **(b)**.

(ii) Use this formula to find A_{200} the number of matches in the 200 th diagram in the sequence.
Expect this to be a lot of matches !

Question 7

A piece of card is $\frac{1}{3}$ of a millimetre thick. i.e. $T_1 = \frac{1}{3}$

(a) It is folded in half, then in half again, and then in half again.
How thick is the folded paper ? i.e. What is T_4 ?

(b) If the paper could be folded twenty-four times in all, how thick would the folded paper be ? i.e. What is T_{25} ?

(c) Is this sequence in Arithmetic Progression ?

(d) What is the formula for the n^{th} term ? i.e. For T_n ?

Question 8

A virus spreads by infecting cells.
Each infected cell infects four uninfected cells each day.

To begin with, at the start of the first day two cells are infected.

How many cells are infected, in total, by the start of the fifth day ?