

Lesson 6

Number Sequences : Year 9

6.1 The term-to-term description of a sequence

In the previous Lessons, all of our formulae for sequences have been *position-to-term* descriptions.

For example, if asked to find the tenth term in the sequence $A_n = n^3 + n$ we use the information about its position, the fact it's the 10th term, to calculate the term in that position. Such formulae are often called *the formula for the n^{th} term*.

There is another way of describing sequences.
It's what's called an *iterative* description or, more helpfully, a *term-to-term* description.

6.2 Examples

(a) A sequence has five terms, T_1 , T_2 , T_3 , T_4 and T_5 .

It is described iteratively by the formula;

$$T_1 = 25, \quad T_{n+1} = \sqrt{T_n} - 1$$

Write out the five terms of the sequence.

(b) A sequence has first term 3, and the following *term-to-term* description;

$$H_{n+1} = \frac{12}{H_n}$$

(i) Write out the first few terms of the sequence.

(ii) What is H_{218} ?

6.3 Exercise

Question 1

A sequence is described iteratively as follows,

$$A_1 = 1, \quad A_{n+1} = 2A_n + 1.$$

Write out the first ten terms of the sequence.

Question 2

A sequence is described iteratively as follows,

$$B_1 = 1, \quad B_{n+1} = B_n + 5.$$

- (a) Write out the first ten terms of the sequence.
- (b) Explain why 3745 will not be in this sequence.

Question 3

A famous sequence is described iteratively as follows,

$$F_1 = 1, \quad F_2 = 1, \quad F_{n+1} = F_n + F_{n-1}$$

- (a) Write out the first ten terms of the sequence.
- (b) What is this sequence's famous name ?

Question 4

A sequence is described iteratively as follows,

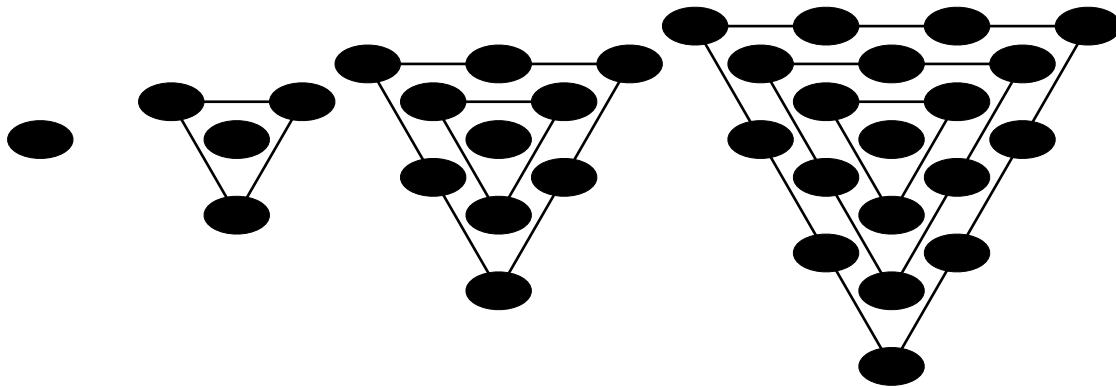
$$S_1 = 1, \quad S_2 = 1, \quad S_{n+1} = S_n - S_{n-1}$$

(a) Write out the first ten terms of the sequence.

(b) What is S_{33} ?

Question 5

Study the sequence of patterns below;



The sequence can be described iteratively;

$$G_1 = 1, \quad G_2 = 4, \quad G_{n+1} = 2G_n - G_{n-1} + 3$$

(a) Show how you would use the information $G_1 = 1$ and $G_2 = 4$ and the iterative relationship to calculate the value of G_3 .

(b) Show how you would use the iterative description to calculate G_4 .

(c) Write out the sequence starting with G_1 and up to G_{10} .

Question 6

A sequence is described iteratively as follows;

$$J_1 = \frac{1}{8}, \quad J_2 = 4, \quad J_{n+1} = (J_n) \times (J_{n-1})$$

Write out the first ten terms of this sequence.

Question 7

A frog is at the bottom of a well 10 feet deep.

He can jump 3 feet, but every time he jumps 3 feet, he falls back 2 feet.

How many jumps will he have to make to get out of the well ?

Show full working.

Question 8

A sequence has the following *position-to-term* formula;

$$M_n = 2^n - 1$$

(a) Write out the first ten terms of this sequence.

(b) This sequence occurred in one of this exercise's previous questions.
Work out which question by looking at the previous answers.
Hence, write down the *term-to-term* formula for this sequence.

Question 9

The two types of sequence describing formulae looked at are

(i) *Position-to-term* formulae such as $A_n = 2n + 1$

and

(ii) *Term-to-term* formulae such as $A_{n+1} = A_n + 8$

In this question the two ideas are mixed together.

A famous sequence is described as follows;

$$S_1 = 1, \quad S_{n+1} = S_n + 2n + 1$$

(a) Write out the first ten terms of this famous sequence.

(b) What is this sequence's famous name ?

Question 10

In this question it's helpful to avoid writing decimal fractions.

A sequence has first term 1 and the following *term-to-term* formula;

$$W_{n+1} = 1 - \frac{W_n}{2}$$

Write out the first ten terms of the sequence.

Write the terms in the form of $\frac{p}{q}$ fractions, for integer p and q .

This is a hard final question !