Lesson 7

Number Sequences : Year 9

7.1 Summary

In this presentation of ***number sequences*** two methods of describing a sequence where studied:

D1 Position-to-term
D2 Term-to-term

Along the way some famous sequences where encountered:

F1 The EVEN numbers
F2 The ODD numbers
F3 The FIBONACCI numbers
F4 The SQUARE numbers
F5 The TRIANGULAR numbers

And, in particular, **ARITHMETIC PROGRESSIONS**, were featured.

7.1.1 D1 Position-to-term.

If asked to find the tenth term in the sequence $A_n = n^3 + n$ use the information about its position, the fact it's the 10^{th} term, to calculate the term in that position.
Such formulae are often called *the formula for the* n^{th} *term*.

7.1.2 D2 Term-to-term.

If asked to find the fourth term in the sequence $A_1 = 1$, $A_{n+1} = 3A_n + 1$, use the first term to get the second, then the second to get the third, and then the third to get the fourth.
This is also referred to as the *iterative* description of a sequence.

7.1.3 F1 The EVEN numbers.

2, 4, 6, 8, 10, ... **Arithmetic Progression** $a = 2$, $d = 2$.

Position-to-term : $E_n = 2n$

Term-to-term : $E_1 = 2$, $E_{n+1} = E_n + 2$

7.1.4 F2 The ODD numbers.

1, 3, 5, 7, 9, ... **Arithmetic Progression** $a = 1$, $d = 2$.

Position-to-term : $O_n = 2n - 1$

Term-to-term : $O_1 = 1$, $O_{n+1} = O_n + 2$

7.1.5 F5 The FIBONACCI numbers.

1, 1, 2, 3, 5, 8, 13, 21, ...

Position-to-term : $F_n = \frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right)^n - \frac{1}{\sqrt{5}}\left(\frac{1-\sqrt{5}}{2}\right)^n$

Term-to-term : $F_{n+1} = F_n + F_{n-1}$

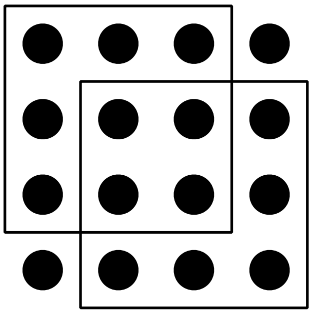
7.1.6 F3 The SQUARE numbers.

1, 4, 9, 16, 25, 36, ...

Position-to-term : $S_n = n^2$

Term-to-term : $S_{n+1} = (\sqrt{S_n} + 1)^2$

$S_{n+1} = 2\,S_n - S_{n-1} + 2$



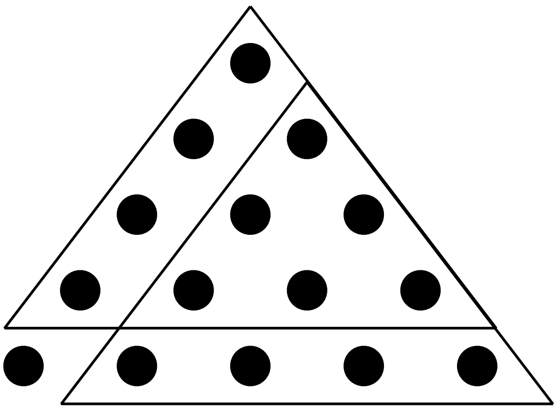
7.1.7 F4 The TRIANGULAR numbers.

1, 3, 6, 10, 15, 21, 28, 36, ...

Position-to-term : $T_n = \frac{1}{2}\, n\,(n + 1)$

Term-to-term : $T_{n+1} = T_n + \frac{1}{2} \sqrt{8\,T_n + 1} + \frac{1}{2}$

$T_{n+1} = 2\,T_n - T_{n-1} + 1$



7.1.8 Arithmetic Progressions.

In algebra, **ALL** arithmetic progressions are of the form;

$a,\ \ a + d,\ \ a + 2\,d,\ \ a + 3\,d,\ \ ...,\ \ a + (n - 1)\,d$

where *a* is *the initial term* and *d* is *the common difference*.

The *position-to-term* formula the n^{th} term of an Arithmetic Progression is :

$$A_n = d\,n + (a - d)$$

The iterative, *term-to-term formula*, for an Arithmetic progression is :

$$A_1 = a, \qquad A_{n+1} = A_n + d$$

7.2 Exercise

Question 1

Fill in the blank cells with the first four terms of each sequence.

Name	1st term	2nd term	3rd term	4th term	*Position-to-term* or *term-to-term* formula
A					$A_n = 4n + 6$
B					$B_n = 13 - 5n$
C					$C_n = (n + 2)(n + 1)$
D					$D_1 = 2, \quad D_{n+1} = D_n + 12$
E					$E_1 = 2, \quad E_{n+1} = 3E_n - 1$
F					$F_1 = 1, \quad F_{n+1} = (\sqrt{F_n} + 1)^2$
G					$G_n = n^2 + 3.5$
H					$H_n = \frac{24}{n} - n$
I					$I_1 = 8, \quad I_{n+1} = \frac{1}{2} I_n - 1$
J					$J_1 = 24, \quad J_{n+1} = \frac{48}{J_n} + 4$

Question 2

Consider the following arithmetic progression;
7, 11, 15, 19, ...

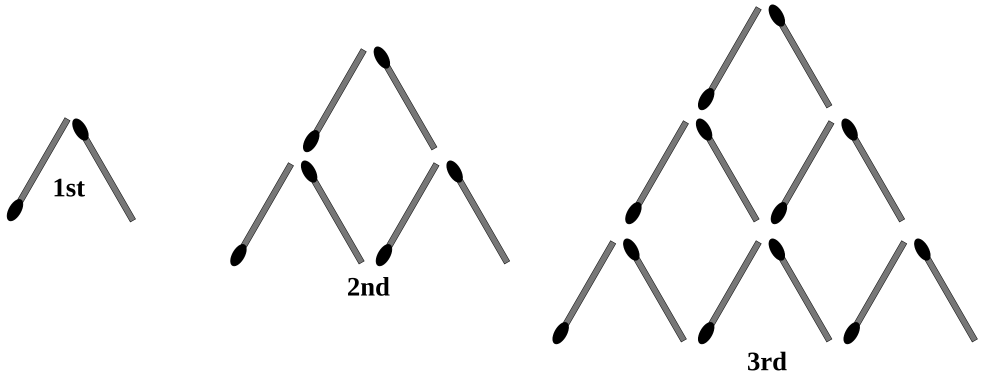
(a) Write down the next three terms.

(b) Write down the formula for the n^{th} term.

(c) Use your formula to determine the 400^{th} term in the sequence.

(d) Could 26788 be a number in this sequence ?
Explain your answer.

Question 3



(a) How many matches will be in the 4 th, 5 th and 6 th diagrams ?

(b) There is a connection with triangular numbers.
Look at section **5.1.6** on triangular numbers for a helpful formula.
Hence write down the *position-to-term* formula for this matchstick sequence.

(c) Use your formula to work out how many matches are needed to make the 100 th matchstick diagram.

Question 4

Complete the following table where each sequence is in Arithmetic Progression.

Name	1st term	2nd term	3rd term	4th term	Initial term a	Common difference d	*Position-to-term* formula $A_n = d\,n + (a - d)$
A	9	15	21	27			$A_n =$
B	- 3	1					$B_n =$
C			17	21			$C_n =$
D					8	3	$D_n =$
E	29		15				$E_n =$
F							$F_n = 7\,n + 15$
G		23				6	$G_n =$
H			28		8		$H_n =$
I							$I_n = 23 - 7\,n$
J		11				9	$J_n =$

Question 5

A sequence has the following *position-to-term* formula;

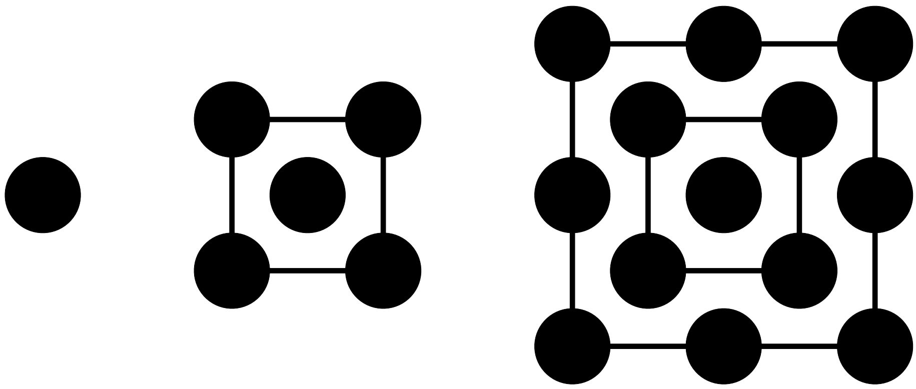
$$M_n = 2^n + 1$$

(a) Write out the first ten terms of this sequence.

(b) Could 34581 be in this sequence ?
Explain your answer.

Question 6

Study the sequence of patterns below;



The sequence can be described iteratively;

$$H_1 = 1, \quad H_2 = 5, \quad H_{n+1} = 2H_n - H_{n-1} + 4,$$

(a) Show how you would use the information that $H_1 = 1$ and $H_2 = 5$ and the iterative relationship to calculate the value of H_3.

(b) Show how you would use the iterative description to calculate H_4.

(c) Write out the sequence starting with H_1 and up to H_{10}.

Question 7

A sequence is described iteratively as follows;

$$J_1 = 2, \qquad J_2 = 16, \qquad J_{n+1} = \frac{J_n}{J_{n-1}}$$

(a) Write out the first ten terms of this sequence.

(b) What is J_{36} ?