

## Lesson 8

## Number Sequences : Year 9

### 8.1 TEST

*You do not need a calculator to do this test but may use one if you wish.*

#### Question 1

Fill in the blank cells with the first four terms of each sequence.

| Name | 1 <sup>st</sup> term | 2 <sup>nd</sup> term | 3 <sup>rd</sup> term | 4 <sup>th</sup> term | Position-to-term or term-to-term formula |
|------|----------------------|----------------------|----------------------|----------------------|------------------------------------------|
| A    |                      |                      |                      |                      | $A_n = 8n + 3$                           |
| B    |                      |                      |                      |                      | $B_n = 9 - 4n$                           |
| C    |                      |                      |                      |                      | $C_n = (n + 3)(n - 1)$                   |
| D    |                      |                      |                      |                      | $D_1 = 5, D_{n+1} = D_n + 3$             |
| E    |                      |                      |                      |                      | $E_1 = 2, E_{n+1} = (E_n)^2$             |
| F    |                      |                      |                      |                      | $F_1 = 0, F_{n+1} = \frac{1}{2}F_n + 8$  |
| G    |                      |                      |                      |                      | $G_n = n^2 - 0.5$                        |
| H    |                      |                      |                      |                      | $H_n = \frac{12}{n} + n$                 |
| I    |                      |                      |                      |                      | $I_1 = 1, I_{n+1} = 5 - 3I_n$            |
| J    |                      |                      |                      |                      | $J_1 = -2, J_{n+1} = 3J_n + 5$           |

[ 20 marks ]

#### Question 2

Consider the following sequence, A, which is in arithmetic progression;

9, 14, 19, ...

(a) Write down the next four terms.

[ 2 marks ]

(b) Write down the formula for  $A_n$  the  $n^{\text{th}}$  term.

[ 3 marks ]

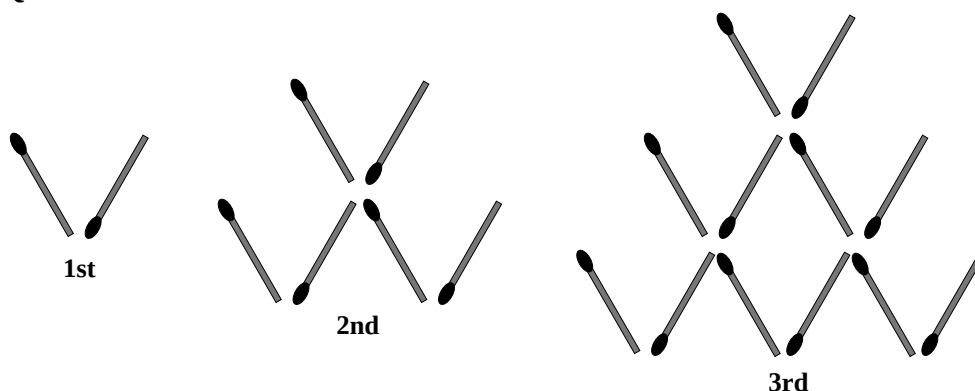
(c) Use your formula to determine the 400<sup>th</sup> term in the sequence.

[ 2 marks ]

(d) Is 26789 a number in this sequence ?  
Explain your answer.

[ 3 marks ]

### Question 3



- ( a ) How many matches will be in the 4<sup>th</sup>, 5<sup>th</sup> 6<sup>th</sup> and 7<sup>th</sup> diagrams ?

[ 4 marks ]

- ( b ) There is a connection with triangular numbers,  $T_n = \frac{1}{2} n ( n + 1 )$   
Hence write down the *position-to-term* formula for this matchstick sequence.

[ 4 marks ]

- ( c ) Use your formula to work out how many matches are needed to make the 100<sup>th</sup> matchstick diagram.

[ 2 marks ]

### Question 4

Complete the following table in which each sequence is in Arithmetic Progression.

| Name | 1 <sup>st</sup> term | 2 <sup>nd</sup> term | 3 <sup>rd</sup> term | 4 <sup>th</sup> term | Initial term<br>$a$ | Common difference<br>$d$ | <i>Position-to-term</i> formula<br>$A_n = d n + (a - d)$ |
|------|----------------------|----------------------|----------------------|----------------------|---------------------|--------------------------|----------------------------------------------------------|
| A    | 10                   | 16                   | 22                   | 28                   |                     |                          | $A_n =$                                                  |
| B    | - 5                  | 1                    |                      |                      |                     |                          | $B_n =$                                                  |
| C    |                      |                      | 29                   | 38                   |                     |                          | $C_n =$                                                  |
| D    |                      |                      |                      |                      | 9                   | 4                        | $D_n =$                                                  |
| E    | 39                   |                      | 21                   |                      |                     |                          | $E_n =$                                                  |
| F    |                      |                      |                      |                      |                     |                          | $F_n = 8 n + 11$                                         |
| G    |                      | 23                   |                      |                      |                     | - 4                      | $G_n =$                                                  |
| H    |                      |                      | 2.8                  |                      | 0.8                 |                          | $H_n =$                                                  |
| I    |                      |                      |                      |                      |                     |                          | $I_n = 17 - 9 n$                                         |
| J    |                      | 11                   |                      | 29                   |                     |                          | $J_n =$                                                  |

[ 20 marks ]

### Question 5

The first five terms of the Fibonacci sequence are;

$$F_1 = 1, \quad F_2 = 1, \quad F_3 = 2, \quad F_4 = 3, \quad F_5 = 5$$

( a ) Write out the next five terms.

$$F_6 = \quad, \quad F_7 = \quad, \quad F_8 = \quad, \quad F_9 = \quad, \quad F_{10} =$$

[ 3 marks ]

( b ) Work out each of the following;

( i )  $F_1 \times F_3 - 1 =$

( ii )  $F_2 \times F_4 + 1 =$

( iii )  $F_3 \times F_5 - 1 =$

( iv )  $F_4 \times F_6 + 1 =$

( v )  $F_5 \times F_7 - 1 =$

( vi )  $F_6 \times F_8 + 1 =$

( vii )  $F_7 \times F_9 - 1 =$

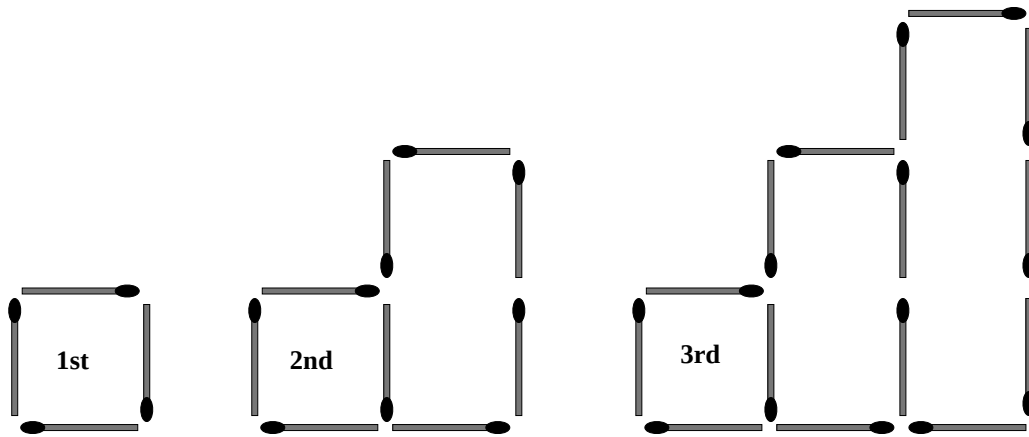
( viii )  $F_8 \times F_{10} + 1 =$

[ 3 marks ]

( c ) Comment on your part ( b ) pattern.

[ 4 marks ]

### Question 6



This matchstick sequence can be described iteratively;

$$K_1 = 4, \quad K_2 = 9, \quad K_{n+1} = 2K_n - K_{n-1} + 1,$$

- (a) Show how you would use the information that  $K_1 = 4$  and  $K_2 = 9$  and the iterative relationship to calculate the value of  $K_3$ .

[ 3 marks ]

- (b) Show how you would use the iterative description to calculate  $K_4$ .

[ 3 marks ]

- (c) Write out the sequence starting with  $K_1$  and up to  $K_{10}$ .

[ 4 marks ]

### Question 7

A sequence is described iteratively as follows;

$$J_1 = 3, \quad J_2 = 15, \quad J_{n+1} = \frac{J_n}{J_{n-1}}$$

(a) Write out the first ten terms of this sequence.

[ 4 marks ]

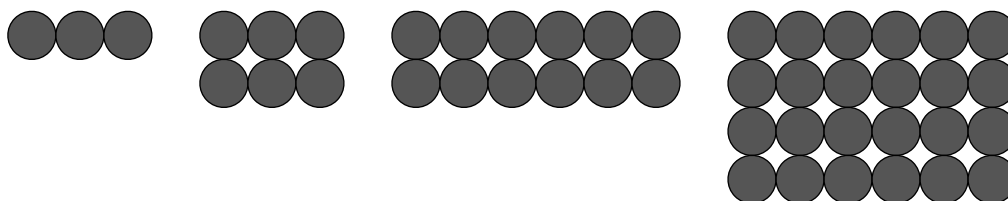
(b) If  $J_1 = n$ ,  $J_2 = m$ , write out the first ten terms of the sequence.  
What have you proved ?

[ 6 marks ]

### Question 8

On a computer three small circles are drawn.

Using *copy and paste* the number of dots grows as shown;



The sequence is described by the formula;

$$D_n = 1.5 \times 2^n$$

(a) Write out the start of this number sequence in the following table;

| Position | 1 <sup>st</sup> | 2 <sup>nd</sup> | 3 <sup>rd</sup> | 4 <sup>th</sup> | 5 <sup>th</sup> | 6 <sup>th</sup> |
|----------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| Term     |                 |                 |                 |                 |                 |                 |

[ 4 marks ]

(b) How would you find  $D_{10}$

(i) Without a calculator ?

[ 3 marks ]

(ii) With a calculator ?

[ 3 marks ]