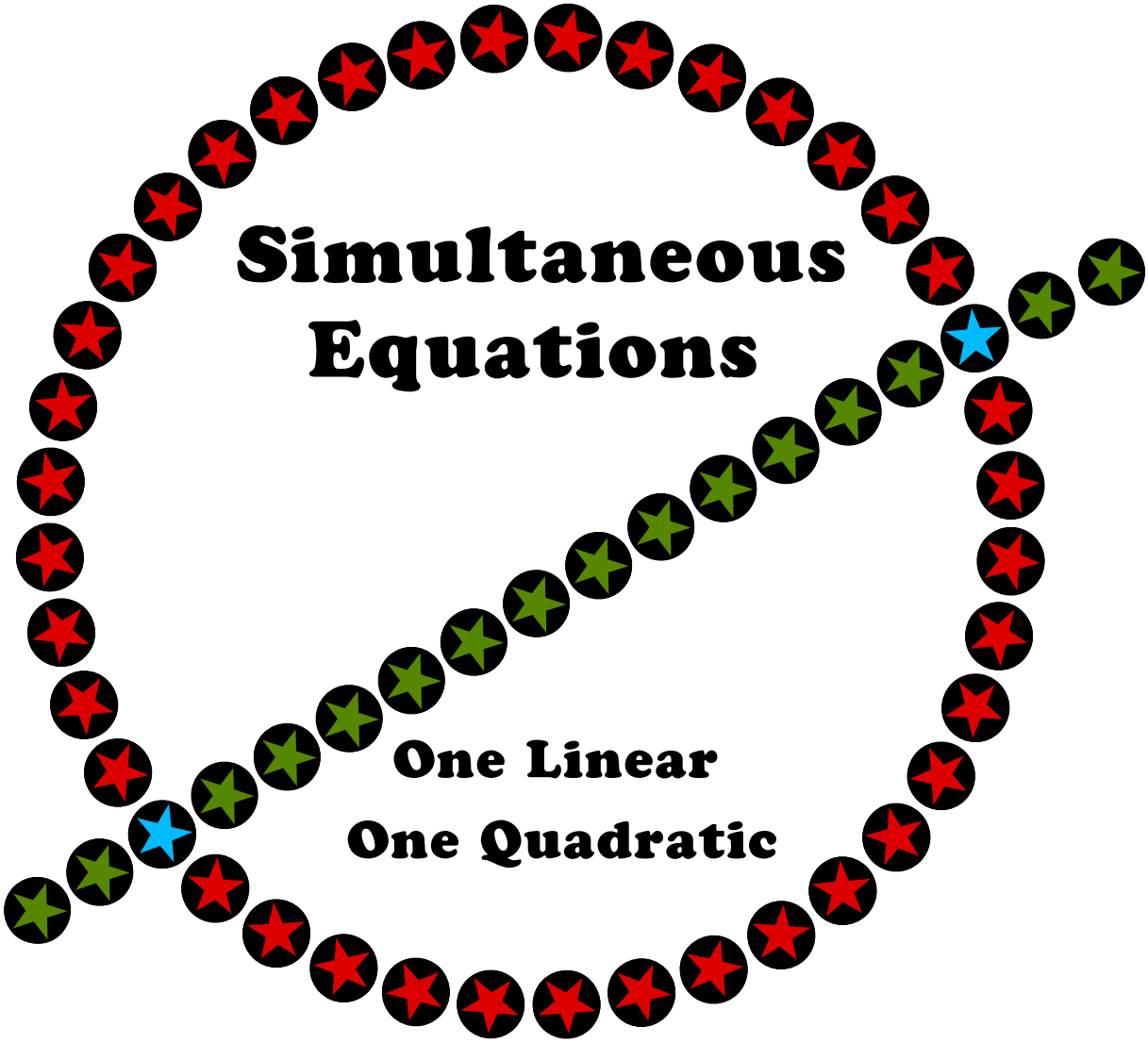
Simultaneous
Equations
One Linear
One Quadratic

Simultaneous Equations

One Linear and One Quadratic

Lesson 1

GCSE Mathematics

Simultaneous Equations III

1.1 Introduction

Previously, we have looked at solving simultaneous equations where both equations were linear. That is, they were *line like*. Geometrically, this equivalent to finding the crossing point of two straight lines.

Now we move on to the situation where one of the two equations is quadratic.

This could be a circle, for example;

$$x^2 + y^2 = 64$$

or a parabola, for example;

$$y = x^2 + 2x + 1$$

What follows next is an example of an examination question. We're not going to solve those yet. Rather we look at what algebraic techniques, studied previously, will be involved in solving such a question.

GCSE, June 2011, paper 3H, Q22

Solve the simultaneous equations,

$$y = 2x - 3 \qquad (\,line\,)$$

$$x^2 + y^2 = 2 \qquad (\,circle\,)$$

Geometrically, this is asking for the points (if any) where the line cuts the circle. Typically there will be two points but there could be one or none.

This topic is really five previous topics glued together.

- Expanding brackets, FOIL
- Gathering together like terms
- Rearranging equations into the form $f(x) = 0$
- Factorising quadratics
- Solving quadratic equations

We'll revise each of these separately, then look at how to glue the bits together.

In an examination, a *solve these simultaneous equations question* is worth about six marks. The bits glued together will also be tested separately elsewhere in the exam. So, being good at this topic is potentially worth about 6% in each of the two examination papers.

1.2 FOIL

This is a first and crucial step. If you get this wrong, the mark scheme may state that no marks are to be awarded. So an error here is 'game over': *Go direct to Jail : Do not pass Go, Do not collect £200.*

Questions

Expand the brackets;

(i) $(x+6)(x+5)$

(ii) $(x+1)(4x+7)$

(iii) $(2x+5)(3x+4)$

(iv) $(8x+5)(11x+9)$

(v) $(x+8)(x-3)$

(vi) $(x-5)(3x+4)$

(vii) $(x-7)(5x-2)$

(viii) $(x+5)^2$

(ix) $(2x+7)^2$

(x) $(5x-3)^2$

The last three questions, where the bracket is squared, are the most important.

1.3 Gathering together like terms

Example

Simplify $3x^2 - 3 + 4x + 2x + x^2 + 7$

Try this, then check with the answer at the bottom of the following page.

Questions

Simplify each of the following;

(i) $x^2 + 5x + 7x + 11$

(ii) $2x^2 + 11x - 6x + 3$

(iii) $5x^2 + 4x + 3 + 5x - 2$

(iv) $x^2 + 4x - 7 - 9x + 3$

(v) $9x^2 + 13x - 6x^2 + 3 - x$

(vi) $3x^2 + 14x + (5x - 3) - x^2$

(vii) $5x^2 - 7x - (4x + 3)$ Be careful expanding the brackets !

(viii) $x^2 + 7x + (7 - 5x) - 3$

(ix) $8x^2 + 14x - \left(4x^2 - 3x\right) - x^2$

(x) $4\left(3x^2 + 2x\right) - (7 - 5x) - 1$

1.4 Rearranging equations into the form $f(x) = 0$

Example

Rearrange the following equation into the form $f(x) = 0$

$5x^2 + 4x + 8 = 2x + 5$

Try this, then check with the answer at the bottom of the following page.

Questions

Rearrange each of the following equation into the form $f(x) = 0$

(i) $8x^2 + 9x + 5 = 4x + 2$

(ii) $2x^2 + 3x + 11 = 7x + 3$

(iii) $7x^2 - 5x + 1 = 3x - 7$

(iv) $9x^2 + 7x - 8 = 5 - 3x$

Previous page answer: $4x^2 + 6x + 4$

(v) $8x^2 + 14x - 8 = 2x^2 - 3x + 2$

(vi) $2x^2 + 15x - 4 = 2x^2 + 3x + 28$

(vii) $5x^2 + (6x - 5) = 9 - 2x + x^2$

(viii) $11x^2 - (4x + 3) = 5x^2 - 15$

(ix) $8x^2 - (7x - 6) = 5 - 3x$

(x) $7x^2 - (3 + 7x) = 2x^2 - (6x - 1)$

1.5 Mixture

Before tackling the last two items on the introduction's lists (next lesson) here are some question that mix the first three ideas.

Questions

Rearrange each of the following equation into the form $f(x) = 0$

(i) $(x + 4)(x + 6) = 5x + 3$

(ii) $(3x + 9)(2x + 3) = 8x - 13$

(iii) $(x + 5)(x - 5) = 7x + 13$

(iv) $(x + 4)(3x - 5) = 2x^2 - 5$

(v) $(x + 7)^2 = 4x + 7$

(vi) $(2x - 3)^2 = (x + 2)^2$

Previous page answer: $5x^2 + 2x + 3 = 0$