4.1 Solving Quadratic Equations

A quadratic equation is an equation of the form,

$$y = ax^2 + bx + c$$

where a, b and c are constants, and x and y are variables.

Last lesson, a large number of quadratic expressions where factorised. The ability to factorise a quadratic expression is one of the key steps in how the quadratic equations in this lesson may be solved.

We are particularly interested in finding any value of x that make $y = 0$. This is because zero in mathematics is a very special number, not least because of the following theorem, The Product of Zero Theorem.

The Product of Zero Theorem

Given two numbers which have a product of zero,
either the first number or the second number must be zero.

$$\text{If } \; p \times q = 0$$

$$\text{Then either } \; p = 0 \text{ or } q = 0$$

The word "or" means either $p = 0$ or $q = 0$ or both $p = 0$ and $q = 0$

It is called an "inclusive or".

(FYI : "Exclusive or", *xor*, excludes the possibility that both are zero)

Note that it is only when $p \times q = 0$ that you get such insightful information about p or q. To see this consider the situation where $p \times q = 24$. Solutions to this, even just in positive integers, include (1, 24), (2, 12), (3, 8), (4, 6), (6, 4), (8, 3), (12, 2), (24, 1). The constraints upon p or q when the product is zero are very strong, which is otherwise not so.

4.2 Example

The Question : Solve, $x^2 + 6x + 8 = 0$

The Answer :

$x^2 + 6x + 8 = 0$	
$(x + 4)(x + 2) = 0$	Factorise
Either $x + 4 = 0$ or $x + 2 = 0$	The Product of Zero Theorem
$\therefore \; x = -4$ or $x = -2$	The "roots" of the quadratic

Notice : Each of the two answers could be substituted back into the quadratic equation to check they are correct.

4.3 Exercise

Question 1

Solve these quadratic equations. Every solution should use the word “either” and the word “or” at the appropriate step.

(i) $x^2 + 8x + 15 = 0$

(ii) $x^2 + 5x + 4 = 0$

(iii) $x^2 + 9x + 14 = 0$

(iv) $x^2 + 9x + 18 = 0$

(v) $x^2 + 15x + 44 = 0$

(vi) $x^2 + 11x + 24 = 0$

(vii) $3x^2 + 33x + 90 = 0$

Hint : Start by dividing through by 3

(viii) $10x^2 + 120x + 270 = 0$

(ix) $4x^2 + 60x + 200 = 0$

(x) $5x^2 + 60x + 160 = 0$

4.4 Example

The Question: Solve, $x^2 - 6x - 55 = 0$

The Answer :

$x^2 - 6x - 55 = 0$	
$(x - 11)(x + 5) = 0$	Factorise
Either $x - 11 = 0$ or $x + 5 = 0$	The Product of Zero Theorem
$\therefore\ x = 11$ or $x = -5$	The “roots” of the quadratic

Be Careful : By far the most common mistake is to get the signs of the answers wrong; (x – 11) is a factor of the quadratic, and $x = 11$ is a root.

4.5 Exercise

Question 1

Solve these quadratic equations. Every solution should use the word “either” and the word “or” at the appropriate step.

(i) $x^2 - 5x - 14 = 0$

(ii) $x^2 - 4x - 21 = 0$

(iii) $x^2 - 10x - 24 = 0$

(iv) $x^2 - x - 30 = 0$

(v) $x^2 - 6x - 40 = 0$

(vi) $x^2 + 6x - 16 = 0$

(vii) $3x^2 + 24x - 99 = 0$

(viii) $7x^2 + 35x - 350 = 0$

(ix) $2x^2 + 32x - 34 = 0$

(x) $3x^2 + 9x - 120 = 0$

(xi) $16x^2 + 256x - 272 = 0$

(xii) $15x^2 + 45x - 600 = 0$