4.1 Solving Quadratic Equations

A quadratic equation is an equation of the form,

$$y = ax^2 + bx + c$$

where a, b and c are constants, and x and y are variables.

Last lesson, a large number of quadratic expressions where factorised. The ability to factorise a quadratic expression is one of the key steps in how the quadratic equations in this lesson may be solved.

We are particularly interested in finding any value of x that make y = 0. This is because zero in mathematics is a very special number, not least because of the following theorem, The Product of Zero Theorem.

The Product of Zero Theorem

Given two numbers which have a product of zero, either the first number or the second number must be zero.

If
$$p \times q = 0$$

Then either p = 0 or q = 0

The word "or" means either p = 0 or q = 0 or both p = 0 and q = 0It is called an "inclusive or".

(FYI: "Exclusive or", xor, excludes the possibility that both are zero)

Note that it is only when $p \times q = 0$ that you get such insightful information about p or q. To see this consider the situation where $p \times q = 24$. Solutions to this, even just in positive integers, include (1, 24), (2, 12), (3, 8), (4, 6), (6, 4), (8, 3), (12, 2), (24, 1). The constraints upon p or q when the product is zero are very strong, which is otherwise not so.

4.2 Example

The Question : Solve, $x^2 + 6x + 8 = 0$

The Answer:

$$x^2 + 6x + 8 = 0$$

 $(x + 4)(x + 2) = 0$ Factorise
Either $x + 4 = 0$ or $x + 2 = 0$ The Product of Zero Theorem
 $\therefore x = -4$ or $x = -2$ The "roots" of the quadratic

Notice: Each of the two answers could be substituted back into the quadratic equation to check they are correct.

4.3 Exercise

Question 1

Solve these quadratic equations. Every solution should use the word "either" and the word "or" at the appropriate step.

(i)
$$x^2 + 8x + 15 = 0$$
 (ii) $x^2 + 5x + 4 = 0$

$$(ii) x^2 + 5x + 4 = 0$$

(iii)
$$x^2 + 9x + 14 = 0$$
 (iv) $x^2 + 9x + 18 = 0$

(iv)
$$x^2 + 9x + 18 = 0$$

$$(\mathbf{v})$$
 $x^2 + 15x + 44 = 0$

$$(\mathbf{v})$$
 $x^2 + 15x + 44 = 0$ (\mathbf{vi}) $x^2 + 11x + 24 = 0$

(vii)
$$3x^2 + 33x + 90 = 0$$
 (viii) $10x^2 + 120x + 270 = 0$

Hint: Start by dividing through by 3

$$(ix)$$
 $4x^2 + 60x + 200 = 0$ (x) $5x^2 + 60x + 160 = 0$

4.4 Example

The Question: Solve,
$$x^2 - 6x - 55 = 0$$

The Answer:

$$x^2 - 6x - 55 = 0$$

 $(x - 11)(x + 5) = 0$ Factorise
Either $x - 11 = 0$ or $x + 5 = 0$ The Product of Zero Theorem
 $\therefore x = 11$ or $x = -5$ The "roots" of the quadratic

Be Careful: By far the most common mistake is to get the signs of the answers wrong; (x-11) is a factor of the quadratic, and x=11 is a root.

4.5 Exercise

Question 1

Solve these quadratic equations. Every solution should use the word "either" and the word "or" at the appropriate step.

(i)
$$x^2 - 5x - 14 = 0$$
 (ii) $x^2 - 4x - 21 = 0$

(iii)
$$x^2 - 10x - 24 = 0$$
 (iv) $x^2 - x - 30 = 0$

$$(\mathbf{v})$$
 $x^2 - 6x - 40 = 0$

$$(\mathbf{v})$$
 $x^2 - 6x - 40 = 0$ (\mathbf{vi}) $x^2 + 6x - 16 = 0$

(vii)
$$3x^2 + 24x - 99 = 0$$

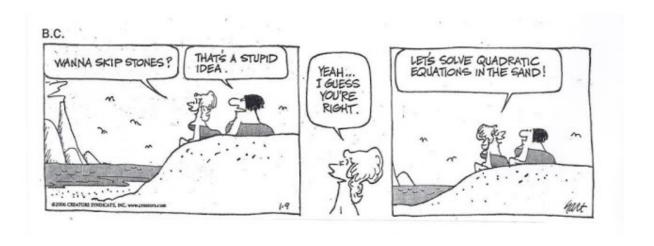
(vii)
$$3x^2 + 24x - 99 = 0$$
 (viii) $7x^2 + 35x - 350 = 0$

(ix)
$$2x^2 + 32x - 34 = 0$$
 (x) $3x^2 + 9x - 120 = 0$

$$(\mathbf{x}) \qquad 3x^2 + 9x - 120 = 0$$

$$(\mathbf{xi}) \qquad 16 \, x^2 + 256x - 272 = 0$$

$$(\mathbf{xii}) \qquad 15\,x^2 + 45x - 600 = 0$$



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 $Teachers\ may\ obtain\ detailed\ worked\ solutions\ to\ the\ exercises\ by\ email\ from\ MHHShrewsbury@Gmail.com$