6.1 Quadratic Equations With Integer Roots (Recap)

Faced with a quadratic equation of the form,

$$y = x^2 + bx + c \qquad \text{where } b \text{ and } c \text{ are integers}$$

we previously, in Lesson 3, developed a fatorisation theory based around the fact that, given the factorisation was of the form,

$$y = (x + A)(x + B) \qquad \text{where } A \text{ and } B \text{ are integers}$$

then,

$$A + B = b \quad \text{and} \quad AB = c$$

Proof:

Given that the quadratics factorise into $(x + A)(x + B)$ means that by expanding the brackets the original quadratic equation must be obtained;

$$(x + A)\ (x + B) = x^2 + Ax + Bx + AB$$

$$= x^2 + (A + B)x + AB$$

Compare this with $y = x^2 + bx + c$ (where the coefficient of x^2 is 1)

$$A + B = b \quad \text{and} \quad AB = c$$

The theory develops further to solve such equations when $y = 0$

$$x^2 + bc + c = 0$$

$$(x + A)(x + B) = 0$$

$$\text{Either} \quad x + A = 0 \qquad \text{or} \quad x + B = 0$$

$$x = -A \qquad\qquad x = -B$$

The solutions to such a quadratic equation are called the roots of the equation.

Example #1

Find the roots of the quadratic equation $y = x^2 - 2x - 3$

6.2 Quadratic Equations With Rational Roots

Faced with a quadratic equation of the form,

$y = a x^2 + bx + c$ where *a, b* and *c* are integers with

$\left(\text{preferably with } hcf\ \{a, b, c\} = 1\right)$

then, if the factorisation is of the form,

$$y = (Px + Q)(Rx + S)$$

then $PR = a$ and $PS + QR = b$ and $QS = c$

Proof:

Given that the quadratics factorise into $(Px + Q)(Rx + S)$ means that by expanding the brackets the original quadratic equation must be obtained;

$$(Px + Q)(Rx + S) = PRx^2 + PSx + QRx + QS$$

$$= PRx^2 + (PS + QR)x + QS$$

Compare this with $y = a x^2 + bx + c$

$PR = a$ and $PS + QR = b$ and $QS = c$

Example #2

Factorise $y = 15x^2 + 11x + 2$

Example #3

Find the roots of the quadratic equation $y = 6x^2 - x - 2$

6.3 Exercise

Question 1

Factorise;

(i) $y = 2x^2 + 13x + 15$

(ii) $y = 9x^2 + 24x + 7$

(iii) $y = 7x^2 + 23x + 6$

(iv) $y = 6x^2 + 25x + 25$

Question 2

Factorise,

(i) $y = 10x^2 - 9x + 2$

(ii) $y = 3x^2 - 7x + 4$

(iii) $y = 6x^2 - 13x + 5$

(iv) $y = 3x^2 - 20x + 33$

Question 3

Factorise;

(i) $y = 5x^2 - 33x - 14$

(ii) $y = 4x^2 - 4x - 3$

(iii) $y = 6x^2 + 7x - 5$

(iv) $y = 14x^2 + 15x - 9$

Question 4

Factorise,

(i) $y = 18x^2 + 27x + 10$

(ii) $y = 12x^2 + 56x + 9$

(iii) $y = 24x^2 + 7x - 6$

(iv) $y = 13x^2 - 43x - 36$

Question 5

Solve these quadratic equations. Every solution should use the word “either” and the word “or” at the appropriate step.

(i) $2x^2 + 7x + 3 = 0$

(ii) $3x^2 + 13x - 10 = 0$

(iii) $4x^2 - 12x - 7 = 0$

(iv) $9x^2 - 38x + 8 = 0$

Question 6

Find the roots of the quadratic equation $y = 12x^2 + 23x + 10$

Question 7

Solve the simultaneous equations,

$$y = 3x^2$$

$$y = 2x + 16$$

Show clear algebraic working.

Question 8

The line with equation $y = \frac{1}{2}x - 3$ intersects the parabola with equation $y = x^2 + 8x - 7$ at the points A and B.

Find the coordinates of A and the coordinates of B.

Show clear algebraic working.

Question 9

Work out the coordinates of the points of intersection of $x - y = 3$ and $y^2 + xy = 5$. Show clear algebraic working.