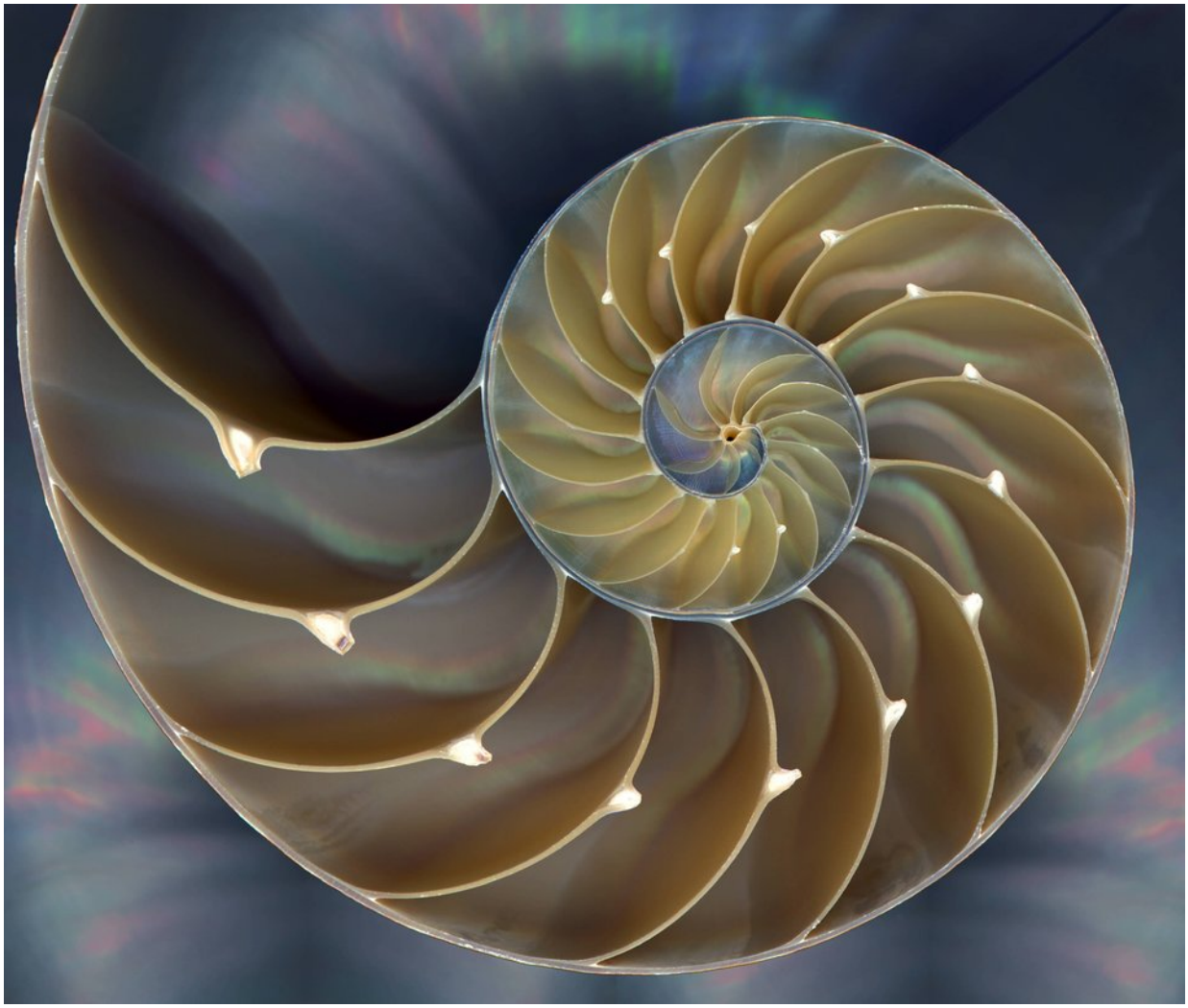
A-Level Pure Mathematics

N U M E R I C A L M E T H O D S

Solving Equations using Iteration



~ Year 2 ~

A-Level Pure Mathematics

N U M E R I C A L M E T H O D S
Solving Equations using Iteration

Lesson 1

Numerical Methods : Pure Year 2

1.1 Solving 'Impossible' Equations

Much of GCSE and A level mathematics is about learning methods for finding the exact solutions to equations, when such solutions exist. However, it is easy to write down an equation for which such methods do not yield solutions, even when it is known that solutions must exist.

For example, consider the following equation;

$$\sin x^c - \ln x = 0$$

It's a job to know where to begin with trying to solve this.
Perhaps it has no solutions, or one, or several, or even an infinite number.

We could make some progress towards deciding how many solutions this equation has (if any) by rearranging it like this;

$$\sin x^c = \ln x$$

and on a graph sketching each of the curves;

$$y = \sin x^c \qquad \text{and} \qquad y = \ln x$$

From the sketch, and perhaps calculating $\ln(2\pi)$, you have a reasoned argument that the equation $\sin x^c - \ln x = 0$ has one solution exactly.

Although we would struggle to solve the equation algebraically, the sketch graph suggests that not only is there one solution but that it lies between 1.5 and 3.5.

In fact, to the nearest integer, the solution is 2.
Here is how we numerically prove this:

When $x = 1.5$:

When $x = 2.5$:

As $\sin x^c - \ln x$ is POSITIVE when $x = 1.5$ and NEGATIVE when $x = 2.5$ there must be a value of x between 1.5 and 2.5 where the equation equals zero exactly.

As all values of x between 1.5 and 2.5 round off to 2, the solution to the nearest integer is 2.

1.2 Exercise

Question 1

$$f(x) = \sin x^c - \ln x$$

Calculate

(i) $f(2.15)$

(ii) $f(2.25)$

Explain how this proves the solution of $f(x) = 0$ correct to one decimal place is 2.2
You'll need to write out the POSITIVE / NEGATIVE argument we used above
and the bit about ROUNDING.

Question 2

$$f(x) = \sin x^c - \ln x$$

Calculate

(i) $f(2.215)$

(ii) $f(2.225)$

(iii) $f(2.235)$

What is the solution of $f(x) = 0$ correct to two decimal places ?

Question 3

$$f(x) = \sin x^c - \ln x$$

Calculate

(i) $f(2.2175)$

(ii) $f(2.2185)$

(iii) $f(2.2195)$

What is the solution of $f(x) = 0$ correct to three decimal places ?

Question 4

$$f(x) = \sin x^c - \ln x$$

What is the solution of $f(x) = 0$ correct to four decimal places ?

Question 5

$$g(x) = x^3 - 5x + 1$$

(a) Show that the equation $g(x) = 0$ has a solution, correct to 1 decimal place of 2.1. Do this by first calculating g (2.05) and g (2.15) and then explain how those answers imply the claimed result.

(b) Improve upon the part **(a)** answer by finding a solution correct to 2 decimal places.
HINT : g (2.105), g (2.115), g (2.125), g (2.135) etc...

Question 6

$$h(x) = x^3 - 4x + 2$$

In trying to quickly locate a solution to $h(x) = 0$ a student calculates $h(0.5)$ and $h(3.5)$ and concludes that there is no root in between.

(i) Calculate $h(0.5)$

(ii) Calculate $h(3.5)$

(iii) Explain why the student thinks there is no root between $x = 0.5$ & $x = 3.5$

(iv) Calculate $h(1.5)$

(v) Explain the significance of your part (iv) answer.

(vi) Find one of the roots of $h(x) = 0$, correct to 2 decimal places.
There are three, but you are only asked to find one.
(Your choice which you decide to work on)