Lesson 8

Numerical Methods : Year 2

8.1 Examination Questions On The Newton-Raphson Iteration Formula

Given an equation of the form

$$f(x) = 0$$

the Newton-Raphson iteration formula to find numerical solutions is

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

8.2 Exercise

Question 1

FP1 Examination Question, June 2011, Q4

$$f(x) = x^2 + \frac{5}{2x} - 3x - 1 \qquad x \neq 0$$

(a) Use differentiation to find $f'(x)$

[2 marks]

The root α of the equation $f(x) = 0$ lies in the interval [0.7, 0.9]

(b) Taking 0.8 as a first approximation to α, apply the Newton-Raphson process once to $f(x)$ to obtain a second approximation to α.
Give your answer to 3 decimal places

[4 marks]

Question 2

FP1 Examination Question, June 2008, Q2

$$f(x) = 4\cos x + e^{-x}$$

(a) Show that the equation $f(x) = 0$ has a root α between 1.6 and 1.7

[2 marks]

(b) Taking 1.6 as your first approximation to α, apply the Newton-Raphson procedure once to $f(x)$ to obtain a second approximation to α.
Give your answer to 3 significant figures

[4 marks]

Question 3

FP1 Examination Question, June 2016, Q2

$$f(x) = 3x^{\frac{3}{2}} - 25x^{-\frac{1}{2}} - 125 \qquad x > 0$$

(a) Find $f'(x)$

[2 marks]

The equation $f(x) = 0$ has a root α in the interval [12, 13]

(b) Using $x_0 = 12.5$ as a first approximation to α, apply the Newton-Raphson process once to $f(x)$ to find a second approximation to α.
Give your answer to 3 decimal places

[4 marks]

Question 4

FP1 Examination Question, June 2017, Q1

$$f(x) = \frac{1}{3}x^2 + \frac{4}{x^2} - 2x - 1 \qquad x > 0$$

(a) Show that the equation $f(x) = 0$ has a root α in the interval [6,7]

[2 marks]

(b) Taking 6 as a first approximation to α, apply the Newton-Raphson process once to $f(x)$ to obtain a second approximation to α.
Give your answer to 2 decimal places

[5 marks]

Question 5

FP1 Examination Question, January 2010, Q2

$$f(x) = x\cos x - 2x + 5$$

(a) Show that $f(x) = 0$ has a root α in the interval [2,2.1]

[2 marks]

(b) Taking 2 as a first approximation to α, apply the Newton-Raphson procedure once to $f(x)$ to obtain a second approximation to α. Give your answer to 2 decimal places

[5 marks]

(c) Show that your answer to part **(b)** gives α **correct** to 2 decimal places

[2 marks]