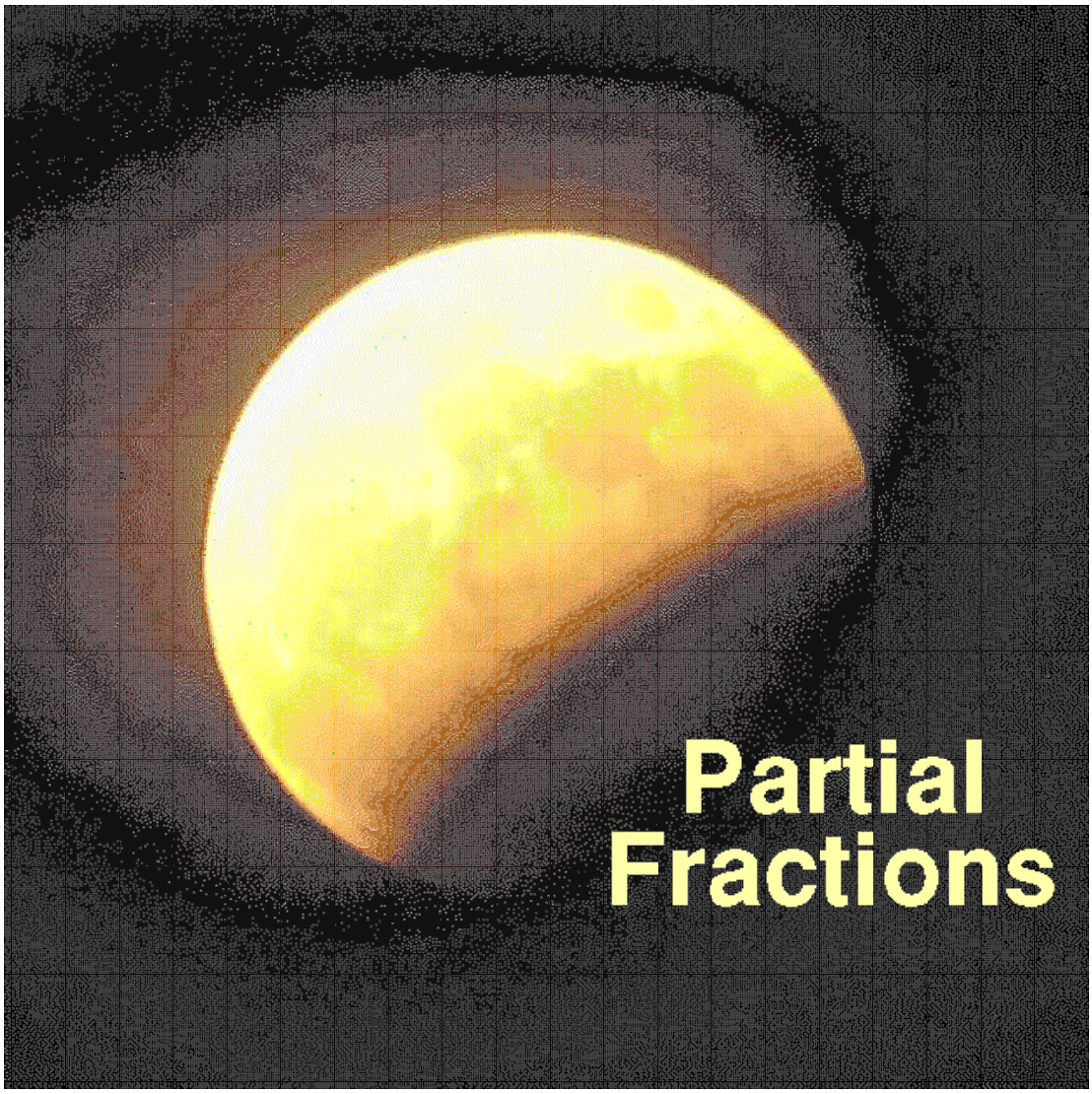
A-Level Pure Mathematics
Year 2

P A R T I A L

F R A C T I O N S



A-Level Pure Mathematics

P A R T I A L F R A C T I O N S
Algebra Skills

Lesson 1

Partial Fractions : Pure Year 2

1.1 Multiplying By 1

Adding vulgar fractions is a skill mastered long ago.
However, let's just look at exactly why we do what we do.

The key idea is that multiplying by 1 is a 'no change' operation.
The number 1 can come in many forms, such as;

$$1 = \frac{5}{5} \qquad \textit{or} \qquad 1 = \frac{11}{11}$$

With that in mind, here is the full detail of how to add a couple of fractions.
The driving strategy is to get a common denominator.

$$\begin{aligned}
\frac{4}{11} + \frac{2}{5} &= \frac{4}{11} \times 1 + \frac{2}{5} \times 1 \\
&= \frac{4}{11} \times \frac{5}{5} + \frac{2}{5} \times \frac{11}{11} \\
&= \frac{4 \times 5}{55} + \frac{2 \times 11}{55} \\
&= \frac{4 \times 5 + 2 \times 11}{55} \\
&= \frac{20 + 22}{55} \\
&= \frac{42}{55}
\end{aligned}$$

The topic of Partial Fractions is about reversing this operation.

So, in the following, the task would be to find the value of the integers A and B.

$$\frac{42}{55} = \frac{A}{11} + \frac{B}{5}$$

You would know to split the denominator of 55 up into 11 and 5 because, when written as a product of primes, $55 = 11 \times 5$.
After multiplying by 55 you'd have the fact that

$$42 = 5A + 11B$$

Solving this in integers would require some 'trial and improvement' but the solution, that $A = 4$ and $B = 2$ is soon found.

1.2 Algebraic Fractions

For GCSE, adding algebraic fractions was tackled as an A/A* grade topic.
A typical question would be to write the following as a single fraction;

$$\frac{3}{(x-4)} - \frac{2}{(x-1)}$$

Again, the key idea is that multiplying by 1 is a 'no change' operation with the number 1 this time being in the forms;

$$1 = \frac{(x-1)}{(x-1)} \qquad \textit{and} \qquad 1 = \frac{(x-4)}{(x-4)}$$

Here is the detail;

$$\begin{aligned}
\frac{3}{(x-4)} - \frac{2}{(x-1)} &= \frac{3}{(x-4)} \times 1 - \frac{2}{(x-1)} \times 1 \\
&= \frac{3}{(x-4)} \times \frac{(x-1)}{(x-1)} - \frac{2}{(x-1)} \times \frac{(x-4)}{(x-4)} \\
&= \frac{3(x-1)}{(x-4)(x-1)} - \frac{2(x-4)}{(x-1)(x-4)} \\
&= \frac{3(x-1) - 2(x-4)}{(x-4)(x-1)} \\
&= \frac{3x - 3 - 2x + 8}{(x-4)(x-1)} \\
&= \frac{x+5}{(x-4)(x-1)}
\end{aligned}$$

1.3 Partial Fractions Exam Style Question

Question : Write the following as partial fractions.

$$\frac{x+5}{(x-4)(x-1)}$$

From section 1.2 we know what the answer is going to be.

That is;

$$\frac{x+5}{(x-4)(x-1)} = \frac{3}{(x-4)} - \frac{2}{(x-1)}$$

But we need to develop a method of obtaining this without such prior knowledge.

1.4 The Square Bracket Method*

We begin by making an assumption about the format of the answer.
Thus;

$$\frac{x+5}{(x-4)(x-1)} = \frac{A}{(x-4)} + \frac{B}{(x-1)}$$

Put square brackets around the question's denominator.

$$\frac{x+5}{[(x-4)(x-1)]} = \frac{A}{(x-4)} + \frac{B}{(x-1)}$$

We're going to multiply through by the square bracket.
However, to lessen the clutter, we're not going to actually write the contents of the square brackets in each time.

$$\frac{(x+5)[\ldots\ldots]}{[(x-4)(x-1)]} = \frac{A[\ldots\ldots]}{(x-4)} + \frac{B[\ldots\ldots]}{(x-1)}$$

With 'virtual cancelling' this becomes;

$$x + 5 = A(x-1) + B(x-4)$$

And this must be true for all values of x.

Let $x = 1$: $\quad 6 = -3B \quad \therefore B = -2$

Let $x = 4$: $\quad 9 = 3A \quad \therefore A = 3$

And so;

$$\frac{x+5}{(x-4)(x-1)} = \frac{3}{(x-4)} - \frac{2}{(x-1)}$$

1.5 Exercise

Question 1
Write the following as partial fractions.

$$\frac{x + 7}{(x - 5)(x + 1)}$$

Question 2
Write the following as partial fractions.

$$\frac{3x + 11}{x^2 + 6x + 5}$$

Question 3

Write the following as partial fractions.

$$\frac{3(3x + 1)}{x^2 - 9}$$

HINT : Difference of two squares.

* This is a method of my own devising.

I find it preferable to 'the cover up rule' as students continue to have an understanding of why it works which is not typically the case with the rival method

1.6 Exercise

Question 1

Write as a single fraction;

$$\frac{3}{(x-2)} + \frac{4}{(x+5)}$$

Question 2

Write as a single fraction;

$$\frac{8}{(x-5)} - \frac{3}{(x+2)}$$

Question 3

Write the following as partial fractions;

$$\frac{3x^2 - 19x + 24}{(x-1)(x-2)(x-3)}$$

Question 4

Write the following as partial fractions;

$$\frac{10}{3x^3 - x^2 - 2x}$$

Question 5

Write the following as partial fractions;

$$\frac{3x}{(x-1)(x+2)(2x-1)}$$

Puzzles
(not too hard)

Question 6

$$\frac{16}{21} = \frac{A}{3} + \frac{B}{7}$$

Find the value of the integers A and B.

Question 7

$$\frac{7}{15} = \frac{A}{3} + \frac{B}{5}$$

Find the value of the integers A and B.