Lesson 3

3.1 Top Heavy Fractions

In numberwork, a "Top Heavy" fraction is one in which the numerator is of greater magnitude that the denominator.

For example,

$$\frac{14}{3}$$

This can handled in a variety of ways, such as,

$$\frac{14}{3} \;=\; 4\frac{2}{3} \;=\; 4.\dot{6}$$

As far as partial fractions are concerned, in algebra, we'd effectively do this;

$$\frac{14}{3} \;=\; 4 \;+\; \frac{2}{3}$$

3.2 Recognizing a Top Heavy Algebraic Fraction

A "Top Heavy" algebraic fraction is one in which the degree of the numerator is equal to, or greater than, the degree of the denominator.

Consider,

$$f(x) \;=\; \frac{x^3 + 4x^2 + 3}{x}$$

The degree of the numerator is 3.
The degree of the denominator is 1.
$\therefore$ It's "Top Heavy".

3.3 Example of Partial Fractions with a Top Heavy Fraction.

Differentiate $f(x)$.

3.4 Analysis of the Example

The crucial step in the example was the 'wedging' of the fraction. That is,

$$\frac{x^3 + 4x^2 + 3}{x} = \frac{x^3}{x} + \frac{4x^2}{x} + \frac{3}{x}$$

$$= x^2 + 4x + \frac{3}{x}$$

So, the form of the Partial Fraction was,

$$\frac{x^3 + 4x^2 + 3}{[x]} = Ax^2 + Bx + C + \frac{D}{x}$$

and the Partial Fraction approach would be: Begin by assuming the solution was of that form and continued by multiplying through by $[x]$.

The partial fraction end point is finding that $A = 1$, $B = 4$, $C = 0$ and $D = 3$.

Of course, such a simple question would not be tackled using Partial Fractions !

3.5 The form of Top Heavy Partial Fractions

Spot the pattern ....

$$\frac{x^4 - 7}{(x-2)(x-3)} = Ax^2 + Bx + C + \frac{D}{(x-2)} + \frac{E}{(x-3)}$$

$$\frac{x^3 - 7}{(x-2)(x-3)} = Ax + B + \frac{C}{(x-2)} + \frac{D}{(x-3)}$$

$$\frac{x^2 - 7}{(x-2)(x-3)} =$$

3.6 Exercise

Question 1

Write as partial fractions,

$$\frac{x^2 - 7}{(x-2)(x-3)}$$

Question 2

Write as partial fractions,

$$\frac{2x^3 + 5}{(x + 1)(x + 2)}$$

Question 3

Write as partial fractions,

$$\frac{x^3 + 12x^2 + 3}{(x - 1)(x + 3)}$$

Question 4

Write as partial fractions,

$$\frac{x^2 + 8x + 4}{(x + 5)(2x - 1)}$$

(Expect some answers to be fractions)