Lesson 4

Partial Fractions : Pure Year 2

4.1 Non-Linear Factors in the Denominator

$$f(x) = \frac{3x + 11}{x^2 + 6x + 5}$$

In order to write $f(x)$ in partial fractions, first factorise the denominator.
Thus;

$$\frac{3x + 11}{x^2 + 6x + 5} = \frac{3x + 11}{(x + 5)(x + 1)}$$

This particular question was tackled in Exercise 1.5, Question 2;

$$\frac{3x + 11}{x^2 + 6x + 5} = \frac{1}{(x + 5)} + \frac{2}{(x + 1)}$$

But what if the quadratic factor could not be factorised ?
For example, $x^2 + x + 2$ cannot be factorised as it has a negative discriminant.

4.2 Example

Write in partial fractions,

$$\frac{8x + 7}{(x^2 + x + 1)(x + 2)}$$

The form of the answer will be;

$$\frac{8x + 7}{(x^2 + x + 1)(x + 2)} = \frac{Ax + B}{(x^2 + x + 1)} + \frac{C}{(x + 2)}$$

(In general the numerator is of degree one less than the denominator)

Now try to determine A, B and C.

4.3 Exercise

Question 1
Write in partial fractions;

$$\frac{3\,(x+2)}{(2x-1)(x^2+1)}$$

Question 2

Write in partial fractions;

$$\frac{6}{(x^2+2)(1-x)}$$

Question 3

(a) Write in partial fractions in the form given;

$$\frac{x^2 + 6x + 4}{(x^3 - 2x)} = \frac{A}{x} + \frac{Bx + C}{x^2 - 2}$$

(b) Write in partial fractions in the form given;

$$\frac{x^2 + 6x + 4}{(x^3 - 2x)} = \frac{A}{x} + \frac{B}{x - \sqrt{2}} + \frac{C}{x + \sqrt{2}}$$